Web Appendix for

On (Un)Congested Roads: A Quantitative Analysis of Infrastructure Investment Efficiency using Truck GPS Data

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B.1. C-Lasso and Penalized GMM

This section outlines the basic idea and the econometric framework for C-Lasso. Su, Shi and Phillips (2016) provide detailed technical derivations, asymptotic properties, and the estimation algorithm.

Consider a linear panel structure model:

$$y_{it} = \varphi_{1,i} + x_{it}\varphi_{2,i} + \mu_i + \varepsilon_{it}, \tag{B.1}$$

where y_{it} is the dependent variable, x_{it} is the independent variable with a size of $1 \times \bar{p}$, $\varphi_{1,i}$ is the intercept parameter, $\varphi_{2,i}$ is a $\bar{p} \times 1$ vector of slope parameters, μ_i is an individual fixed effect, and ε_{it} represents the error term. The demeaned model is

$$\widetilde{y}_{it} = \widetilde{x}_{it}\varphi_{2,i} + \widetilde{\varepsilon}_{it}, \tag{B.2}$$

where T_i is the number of time periods of individual i, $\widetilde{y}_{it} \equiv y_{it} - \frac{1}{T_i}\sum_{t=1}^{T_i} y_{it}$, $\widetilde{x}_{it}$ and $\widetilde{\varepsilon}_{it}$ are defined analogously.

When x_{it} is exogenous, we can use penalized least squares (PLS). The implementation of PLS consists of two parts. First, given the number of groups H and the tuning parameter $\overline{\lambda}$, we have the criteria function

$$Q_{\mathcal{M}T,\bar{\lambda}}^{H,PLS}(\boldsymbol{\varphi_2}, \boldsymbol{\varphi_3}) \equiv \frac{1}{\sum_{i=1}^{\mathcal{M}} T_i} \sum_{i=1}^{\mathcal{M}} \sum_{t=1}^{T_i} (\widetilde{y}_{it} - \widetilde{x}_{it}\varphi_{2,i})^2 + \frac{\bar{\lambda}}{N} \sum_{i=1}^{\mathcal{M}} \prod_{h=1}^{H} \| \varphi_{2,i} - \varphi_{3,h} \|, \tag{B.3}$$

where $\mathcal{M}$ is the number of individuals and $\varphi_{3,h}$ is the group-specific slope parameter. The first part of this criteria function is the same as the objective function in standard fixed effect estimation, and the second component is a variant of standard Lasso. $(\boldsymbol{\varphi_2}, \boldsymbol{\varphi_3})$ can be estimated through the minimization of this criteria function, and based on the estimators we can classify $\mathcal{M}$ individuals into H groups where the individuals in the same group share the same slope parameter. The second step is to decide which H and $\bar{\lambda}$ should be used for our group classification. For each combination of $(H, \bar{\lambda})$, we can not only estimate $(\boldsymbol{\varphi_2}, \boldsymbol{\varphi_3})$ and decide the corresponding group classification, but also calculate an information criteria (IC) value as follows:

$$IC\left(H, \overline{\lambda}\right) = \ln\left\{\frac{1}{\sum_{i=1}^{\mathcal{M}} T_i} \sum_{h=1}^{H} \left[\sum_{i \in \hat{\mathcal{H}}_H\left(H, \overline{\lambda}\right)} \sum_{t=1}^{T_i} \left(\widetilde{y}_{it} - \widetilde{x}_{it}\hat{\varphi}_{3,\hat{\mathcal{H}}_h\left(H,\overline{\lambda}\right)}\right)^2\right]\right\} + \rho_{\mathcal{M}T} \times pH, \tag{B.4}$$

where $\hat{\mathcal{H}}_h\left(H, \bar{\lambda}\right)$ indicates the set of individuals in group h, $\hat{\varphi}_{3,\hat{\mathcal{H}}_h\left(H,\overline{\lambda}\right)}$ is the estimated value of $\varphi_{3,h}$, and the tuning parameter $\rho_{\mathcal{M}T}$ is set to $2/3$ as in Su, Shi and Phillips (2016).

We search over $\left(H, \bar{\lambda}\right)$ to minimize the IC value.

When x_{it} is endogenous, we can introduce instrument variables and then apply penalized GMM (PGMM), which also consists of two parts. The first part is to minimize the criteria function:

$$Q^{H,PGMM}_{\mathcal{M}T,\bar{\lambda}}\left(\boldsymbol{\beta}, \boldsymbol{\varphi_3}\right) \equiv \frac{1}{\mathcal{M}} \sum_{i=1}^{\mathcal{M}} \left\{ \left[\frac{1}{T_i} \sum_{t=1}^{T} \widetilde{z}_{it}' \left(\widetilde{y}_{it} - \widetilde{x}_{it}\varphi_{2,i}\right) \right]' \widetilde{W}_{i,\mathcal{M}T} \left[\frac{1}{T_i} \sum_{t=1}^{T} \widetilde{z}_{it}' \left(\widetilde{y}_{it} - \widetilde{x}_{it}\varphi_{2,i}\right) \right] \right\} + \frac{\bar{\lambda}}{\mathcal{M}} \sum_{i=1}^{\mathcal{M}} \prod_{h=1}^{H} \parallel \beta_i - \varphi_{3,h} \parallel, \tag{B.5}$$

where z_{it} contains both the instrument variables and the exogenous part of x_{it} and $\widetilde{z}_{it} = z_{it} - \frac{1}{T_i}\sum_{t=1}^{T_i} z_{it}$, and $\widetilde{W}_{i,\mathcal{M}T}$ is the weight matrix. The remaining part is to search over $\left(H, \bar{\lambda}\right)$ to minimize the IC value

$$IC\left(H, \bar{\lambda}\right) = \ln\left\{ \frac{1}{\sum_{i=1}^{\mathcal{M}} T_i} \sum_{h=1}^{H} \left[\sum_{i \in \hat{\mathcal{H}}_h(H,\lambda)} \sum_{t=1}^{T_i} \left(\widetilde{y}_{it} - \widehat{\widetilde{x}}_{it} \hat{\varphi}_{3,\hat{\mathcal{H}}_h\left(H,\bar{\lambda}\right)} \right)^2 \right] \right\} + \rho_{\mathcal{M}T} \times pH, \tag{B.6}$$

where $\widehat{\widetilde{x}}_{it}$ is the fitted value of the first stage estimation for each group.

B.2. Derivations of Welfare Elasticities

Elasticity of Welfare with Respect to Expected Trade Cost

In this section, we rephrase the expressions from Allen and Arkolakis (2019) to derive the formula of $d\ln W/d\ln\tau_{kl}$.

Following Allen and Arkolakis (2019), we first express the equilibrium conditions by explicitly incorporating welfare equalization as

$$w_i^{\sigma} L_i^{1-\alpha(\sigma-1)} = W^{1-\sigma} \sum_j \tau_{ij}^{1-\sigma} \bar{A}_i^{\sigma-1} \bar{u}_j^{\sigma-1} w_j^{\sigma} L_j^{1+\beta(\sigma-1)}$$
$$w_i^{1-\sigma} L_i^{\beta(1-\sigma)} = W^{1-\sigma} \sum_j \tau_{ji}^{1-\sigma} \bar{A}_j^{\sigma-1} \bar{u}_i^{\sigma-1} w_j^{1-\sigma} L_j^{\alpha(\sigma-1)}$$

and define

$$x_i \equiv \left(W^{\frac{-1}{\alpha+\beta}} L_i\right)^{\beta(\sigma-1)+1} w_i^{\sigma}$$
$$y_i \equiv \left(W^{\frac{-1}{\alpha+\beta}} L_i\right)^{\alpha(\sigma-1)} w_i^{1-\sigma}$$
$$K_{ij} \equiv \tau_{ij}^{1-\sigma} \bar{A}_i^{\sigma-1} \bar{u}_j^{\sigma-1},$$

which imply the equilibrium conditions can be expressed as

$$
\begin{aligned}
x_i^{\frac{\alpha+1}{\beta(\sigma-1)+1+\alpha\sigma}} y_i^{\frac{-\beta+1}{\beta(\sigma-1)+1+\alpha\sigma}} &= \sum_j K_{ij} x_j y_i \\
x_i^{\frac{\alpha+1}{\beta(\sigma-1)+1+\alpha\sigma}} y_i^{\frac{-\beta+1}{\beta(\sigma-1)+1+\alpha\sigma}} &= \sum_j K_{ji} x_i y_j .
\end{aligned}
$$

Then, by applying the implicit function theorem on equilibrium conditions and normalizing $w_s = 1$,[1] we can get

$$
\begin{aligned}
\frac{d\ln y_i}{d\ln \tau_{kl}} &= (\sigma-1)\left[DGI_{i,k}\frac{X_{kl}}{Y_k} + DGI_{i,N+l}\frac{X_{kl}}{E_l}\right] \\
&= (\sigma-1)\left[DGI_{i,k}\frac{X_{kl}}{Y_k}\right] \; if\ l = s \\
\frac{d\ln x_i}{d\ln \tau_{kl}} &= (\sigma-1)\left[DGI_{N+i,k}\frac{X_{kl}}{Y_k} + DGI_{N+i,N+l}\frac{X_{kl}}{E_l}\right] \\
&= (\sigma-1)\left[DGI_{N+i,k}\frac{X_{kl}}{Y_k}\right] \; if\ l = s,
\end{aligned}
$$

where

$$
DG \equiv \begin{bmatrix} \frac{\sigma(\alpha+\beta)}{\beta(\sigma-1)+1+\alpha\sigma} I & \frac{X}{Y} - \frac{1+\alpha}{\beta(\sigma-1)+1+\alpha\sigma} I \\ \left(\frac{X}{E}\right)' - \frac{1-\beta}{\beta(\sigma-1)+1+\alpha\sigma} I & \frac{(\sigma-1)(\alpha+\beta)}{\beta(\sigma-1)+1+\alpha\sigma} I \end{bmatrix} - \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{X_{1s}}{E_s} & \cdots & \frac{X_{ss}}{E_s} + \frac{\beta}{\alpha} & \cdots & \frac{X_{Ns}}{E_s} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
DGI \equiv DG^{-1}
$$

$$
\frac{X}{Y} \equiv \begin{bmatrix} \frac{X_{11}}{Y_1} & \frac{X_{12}}{Y_1} & \cdots & \frac{X_{1N}}{Y_1} \\ \frac{X_{21}}{Y_2} & \frac{X_{22}}{Y_2} & \cdots & \frac{X_{2N}}{Y_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{X_{N1}}{Y_N} & \frac{X_{N2}}{Y_N} & \cdots & \frac{X_{NN}}{Y_N} \end{bmatrix}, \; \frac{X}{E} \equiv \begin{bmatrix} \frac{X_{11}}{E_1} & \frac{X_{12}}{E_1} & \cdots & \frac{X_{1N}}{E_1} \\ \frac{X_{21}}{E_2} & \frac{X_{22}}{E_2} & \cdots & \frac{X_{2N}}{E_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{X_{N1}}{E_N} & \frac{X_{N2}}{E_N} & \cdots & \frac{X_{NN}}{E_N} \end{bmatrix}
$$

and I indicates the identity matrix.

[1] s indicates the area with the highest GDP.

Finally, based on the definitions of x_i and y_i, we can get

$$\frac{d\ln W}{d\ln\tau_{kl}} = -\frac{2-\rho}{2\sigma-1}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right) + (\alpha+\beta)\frac{d\ln L_i}{d\ln\tau_{kl}}.$$

Further utilizing that $\sum_i L_i = \bar{L}$, we can obtain [2]

$$\begin{aligned}\frac{d\ln W}{d\ln\tau_{kl}} &= -\frac{2-\rho}{2\sigma-1}\sum_i \frac{L_i}{\bar{L}}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right) \\ &= \begin{cases} -X_{kl}\left(1+\varsigma_k+\varpi_l\right) \\ -X_{kl}\left(1+\varsigma_k\right) & if\ l = s, \end{cases}\end{aligned}$$

where

$$\begin{aligned}\rho &\equiv \frac{2+\alpha-\beta}{\beta(\sigma-1)+1+\alpha\sigma} \\ \varsigma_k &\equiv \frac{(\alpha+\beta)(\sigma-1)}{\beta(\sigma-1)+1+\alpha\sigma}\sum_i \frac{L_i}{\bar{L}}\left(DGI_{N+i,k} + \frac{\sigma}{\sigma-1}DGI_{i,k}\right)\frac{1}{Y_k} - 1 \\ \varpi_l &\equiv \frac{(\alpha+\beta)(\sigma-1)}{\beta(\sigma-1)+1+\alpha\sigma}\sum_i \frac{L_i}{\bar{L}}\left(DGI_{N+i,N+l} + \frac{\sigma}{\sigma-1}DGI_{i,N+l}\right)\frac{1}{E_l}.\end{aligned}$$

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[2]

$$\begin{aligned}&\frac{d\ln W}{d\ln\tau_{kl}} = -\frac{2-\rho}{2\sigma-1}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right) + (\alpha+\beta)\frac{d\ln L_i}{d\ln\tau_{kl}} \\ \Longleftrightarrow &\frac{L_i}{\bar{L}}\frac{d\ln W}{d\ln\tau_{kl}} == -\frac{2-\rho}{2\sigma-1}\frac{L_i}{\bar{L}}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right) + (\alpha+\beta)\frac{1}{\bar{L}}\frac{dL_i}{d\ln\tau_{kl}} \\ \Longrightarrow &\frac{d\ln W}{d\ln\tau_{kl}}\sum_i \frac{L_i}{\bar{L}} = -\frac{2-\rho}{2\sigma-1}\sum_i \frac{L_i}{\bar{L}}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right) + (\alpha+\beta)\frac{1}{\bar{L}}\frac{\sum_i dL_i}{d\ln\tau_{kl}} \\ \Longrightarrow &\frac{d\ln W}{d\ln\tau_{kl}} = -\frac{2-\rho}{2\sigma-1}\sum_i \frac{L_i}{\bar{L}}\left(\frac{dx_i}{d\tau_{kl}} + \frac{\sigma}{\sigma-1}\frac{dy_i}{d\tau_{kl}}\right)\end{aligned}$$

Based on the definitions of x_i and y_i, we can get

$$
\begin{aligned}
\frac{d\ln w_i}{d\ln\tau_{kl}} &= \frac{\alpha}{\beta(\sigma-1)+1+\alpha\sigma}\frac{d\ln x_i}{d\ln\tau_{kl}} - \frac{1}{\sigma-1}\frac{\beta(\sigma-1)+1}{\beta(\sigma-1)+1+\alpha\sigma}\frac{d\ln y_i}{d\ln\tau_{kl}} \\
\frac{d\ln L_i}{d\ln\tau_{kl}} &= \frac{1}{\beta(\sigma-1)+1}\frac{d\ln x_i}{d\ln\tau_{kl}} - \frac{\sigma}{\beta(\sigma-1)+1}\frac{d\ln w_i}{d\ln\tau_{kl}} + \frac{1}{\alpha+\beta}\frac{d\ln W}{d\ln\tau_{kl}} \\
\frac{d\ln p_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} - \alpha\frac{d\ln L_i}{d\ln\tau_{kl}} \\
\frac{d\ln P_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} - \frac{d\ln W}{d\ln\tau_{kl}} + \beta\frac{d\ln L_i}{d\ln\tau_{kl}} \\
\frac{d\ln Y_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} + \frac{d\ln L_i}{d\ln\tau_{kl}} \\
\frac{d\ln X_{ij}}{d\ln\tau_{kl}} &= (1-\sigma)\,1\{ij=kl\} + (1-\sigma)\frac{d\ln p_i}{d\ln\tau_{kl}} + (\sigma-1)\frac{d\ln P_j}{d\ln\tau_{kl}} + \frac{d\ln Y_j}{d\ln\tau_{kl}}.
\end{aligned}
$$

Elasticity of Welfare with Respect to Expected Trade Cost without Externality

When there is no externality, the market equilibrium is equivalent to the social planner's solution, and thus we can directly solve for $d\ln W_i/d\ln\tau_{kl}$ through social welfare maximization and the Envelop Theorem

$$\frac{d\ln W}{d\ln\tau_{kl}} = -\frac{X_{kl}}{\sum_i\sum_j X_{ij}}.$$

Some related elasticities can also be derived as follows. The simplified equilibrium conditions become

$$
\begin{aligned}
w_i^{\sigma}L_i &= W^{1-\sigma}\sum_j \tau_{ij}^{1-\sigma}\bar{A}_i^{\sigma-1}\bar{u}_j^{\sigma-1}w_j^{\sigma}L_j \\
w_i^{1-\sigma} &= W^{1-\sigma}\sum_j \tau_{ji}^{1-\sigma}\bar{A}_j^{\sigma-1}\bar{u}_i^{\sigma-1}w_j^{1-\sigma}.
\end{aligned}
$$

Let's define

$$
\begin{aligned}
x_i &\equiv w_i^{\sigma}L_i \\
y_i &\equiv w_i^{1-\sigma} \\
K_{ij} &\equiv \tau_{ij}^{1-\sigma}\bar{A}_i^{\sigma-1}\bar{u}_j^{\sigma-1}
\end{aligned}
$$

and then transform the equilibrium conditions into

$$
\begin{aligned}
x_i &= W^{1-\sigma}\sum_j K_{ij}x_j \\
y_i &= W^{1-\sigma}\sum_j K_{ji}y_j.
\end{aligned}
$$

Then based on the formula of $d\ln W/d\ln\tau_{kl}$ and by applying the implicit function theorem on equilibrium conditions, we can get

$$\left[\frac{X}{Y}-I\right]\begin{bmatrix}\frac{d\ln x_1}{d\ln\tau_{kl}}\\ \frac{d\ln x_2}{d\ln\tau_{kl}}\\ \vdots\\ \frac{d\ln x_N}{d\ln\tau_{kl}}\end{bmatrix}+(\sigma-1)\frac{X_{kl}}{\sum_i\sum_j X_{ij}}\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix}+(1-\sigma)\begin{bmatrix}0\\ \vdots\\0\\ \frac{X_{kl}}{Y_k}\\0\\ \vdots\\0\end{bmatrix}=0$$

$$\left[\left(\frac{X}{E}\right)'-I\right]\begin{bmatrix}\frac{d\ln y_1}{d\ln\tau_{kl}}\\ \frac{d\ln y_2}{d\ln\tau_{kl}}\\ \vdots\\ \frac{d\ln y_N}{d\ln\tau_{kl}}\end{bmatrix}+(\sigma-1)\frac{X_{kl}}{\sum_i\sum_j X_{ij}}\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix}+(1-\sigma)\begin{bmatrix}0\\ \vdots\\0\\ \frac{X_{kl}}{E_l}\\0\\ \vdots\\0\end{bmatrix}=0,$$

where X_{kl}/Y_k is located in the $k-th$ row, and X_{kl}/E_l is located in the $l-th$ row. Because $\left[\left(\frac{X}{E}\right)'-I\right]$ is singular, we need to add a normalization (e.g. $w_s=1\iff y_s=1$) and then obtain

$$\frac{d\ln y_s}{d\ln\tau_{kl}}=0$$

$$\begin{bmatrix}\frac{d\ln y_1}{d\ln\tau_{kl}}\\ \vdots\\ \frac{d\ln y_{s-1}}{d\ln\tau_{kl}}\\ \frac{d\ln y_{s+1}}{d\ln\tau_{kl}}\\ \vdots\\ \frac{d\ln y_N}{d\ln\tau_{kl}}\end{bmatrix}=\left[\left(\frac{X}{E}\right)'-I\right]^{-1}_{-s,-s}\left\{-(\sigma-1)\frac{X_{kl}}{\sum_i\sum_j X_{ij}}\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix}_{-s}-(1-\sigma)\begin{bmatrix}0\\ \vdots\\0\\ \frac{X_{kl}}{E_l}\\0\\ \vdots\\0\end{bmatrix}_{-s}\right\}$$

$$\Rightarrow\frac{d\ln w_i}{d\ln\tau_{kl}}=\frac{1}{1-\sigma}\frac{d\ln y_i}{d\ln\tau_{kl}},$$

where $-s$ indicates drop the $s-th$ row or column. Because $\left[\frac{X}{Y}-I\right]$ is also singular, we first normalize $L_s = 1$ to get

$$\frac{d\ln L_s}{d\ln \tau_{kl}} = 0$$

$$\begin{bmatrix} \frac{d\ln L_1}{d\ln\tau_{kl}} \\ \vdots \\ \frac{d\ln L_{s-1}}{d\ln\tau_{kl}} \\ \frac{d\ln L_{s+1}}{d\ln\tau_{kl}} \\ \vdots \\ \frac{d\ln L_N}{d\ln\tau_{kl}} \end{bmatrix} = \left[\frac{X}{Y} - I\right]_{-s,-s}^{-1} \left\{ -(\sigma-1)\frac{X_{kl}}{\sum_i \sum_j X_{ij}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{-s} - (1-\sigma) \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{X_{kl}}{Y_k} \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{-s} - \sigma \left[\frac{X}{Y} - I\right] \begin{bmatrix} \frac{d\ln w_1}{d\ln\tau_{kl}} \\ \vdots \\ \frac{d\ln w_{s-1}}{d\ln\tau_{kl}} \\ \frac{d\ln w_{s+1}}{d\ln\tau_{kl}} \\ \vdots \\ \frac{d\ln w_N}{d\ln\tau_{kl}} \end{bmatrix} \right\}$$

and then recover the scale of labor elasticity using $\sum_i L_i = \bar{L}$. Thus we can get

$$\frac{d\ln x_i}{d\ln\tau_{kl}} = \sigma\frac{d\ln w_i}{d\ln\tau_{kl}} + \frac{d\ln L_i}{d\ln\tau_{kl}}.$$

Furthermore, we can get

$$\begin{aligned} \frac{d\ln p_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} \\ \frac{d\ln P_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} - \frac{d\ln W}{d\ln\tau_{kl}} \\ \frac{d\ln Y_i}{d\ln\tau_{kl}} &= \frac{d\ln w_i}{d\ln\tau_{kl}} + \frac{d\ln L_i}{d\ln\tau_{kl}} \\ \frac{d\ln X_{ij}}{d\ln\tau_{kl}} &= (1-\sigma)\,1\{ij = kl\} + (1-\sigma)\frac{d\ln p_i}{d\ln\tau_{kl}} + (\sigma-1)\frac{d\ln P_j}{d\ln\tau_{kl}} + \frac{d\ln Y_j}{d\ln\tau_{kl}}. \end{aligned}$$

Elasticity of Welfare with Respect to Expected Trade Cost without Labor Mobility

The model becomes the standard Armington model when there is no labor mobility; we can apply Allen, Arkolakis and Takahashi (2020)'s approach to first get

$$\begin{aligned} \frac{d\ln p_l}{d\ln\tau_{ij}} &= -\phi X_{ij}\left(b_{l,i} + b_{l,N+j}\right) \\ \frac{d\ln P_l}{d\ln\tau_{ij}} &= -\phi X_{ij}\left(b_{N+l,i} + b_{N+l,N+j}\right), \end{aligned}$$

where p_s is normalized to 1, $\phi \equiv \sigma - 1$, $\psi \equiv -\frac{\alpha+1}{\alpha+\beta}$, and b_{ij} is the $i-jth$ element of

$$A^{-1} \equiv \begin{bmatrix} (1+\psi)\, diag\,(Y) - (1+\psi)\, X + \phi diag\,(Y) & -\psi diag\,(Y) - (\phi - \psi)\, X \\ \phi X' & -diag\,(Y) \end{bmatrix}^{-1}.$$

Then, for the welfare we have

$$\begin{aligned} W_i &= \frac{w_i}{P_i} = \frac{p_i A_i}{P_i} \\ \Rightarrow \frac{d\ln W_i}{d\ln \tau_{kl}} &= \frac{d\ln p_i}{d\ln \tau_{kl}} - \frac{d\ln P_i}{d\ln \tau_{kl}} \end{aligned}$$

and

$$\begin{aligned} \frac{d\ln L_i}{d\ln \tau_{kl}} &= 0 \\ \frac{d\ln w_i}{d\ln \tau_{kl}} &= \frac{d\ln p_i}{d\ln \tau_{kl}} \\ \frac{d\ln Y_i}{d\ln \tau_{kl}} &= \frac{d\ln w_i}{d\ln \tau_{kl}} \\ \frac{d\ln X_{ij}}{d\ln \tau_{kl}} &= (1-\sigma)\, 1\{ij = kl\} + (1-\sigma) \frac{d\ln p_i}{d\ln \tau_{kl}} + (\sigma - 1) \frac{d\ln P_j}{d\ln \tau_{kl}} + \frac{d\ln Y_j}{d\ln \tau_{kl}}. \end{aligned}$$

Recall that in the calculation of $d\ln W_i / d\ln \tau_{kl}$ above, we normalize $w_s = 1$ when using Allen and Arkolakis (2019)'s formula, but normalize $p_s = 1$ when using Allen, Arkolakis and Takahashi (2020)'s formula. As $w_s = p_s / A_s$, and there is no labor mobility and externality now, these two versions of normalization are equivalent.

Elasticity of Trade Flow with Respect to Direct Trade Cost

Recall that $\Xi_{kl} = \sum_i \sum_j X_{ij} \left[\tau_{ij} / \left(\tau_{ik} t_{kl} \tau_{lj}\right)\right]^{\theta}$. The elasticity of Ξ_{kl} with respect to t_{kl} is

$$\begin{aligned} \frac{d\ln \Xi_{ij}}{d\ln t_{kl}} &= \frac{1}{\sum_{m'n'} X_{m'n'} \pi^{ij}_{m'n'}} \sum_{mn} \left(\frac{dX_{mn}}{d\ln t_{kl}} \pi^{ij}_{mn} + X_{mn} \frac{d\pi^{ij}_{mn}}{d\ln t_{kl}} \right) \\ &= \frac{1}{\sum_{m'n'} X_{m'n'} \pi^{ij}_{m'n'}} \sum_{mn} X_{mn} \pi^{ij}_{mn} \left[\sum_{k'l'} \frac{d\ln X_{mn}}{d\ln \tau_{k'l'}} + \theta \left(\pi^{kl}_{mn} - \pi^{kl}_{mi} - \pi^{kl}_{jn} - 1\{ij = kl\} \right) \right], \end{aligned}$$

where $dX_{mn} / d\ln \tau_{k'l'}$ is obtained in Section A.2 and $\pi^{kl}_{mn} \equiv d\ln \tau_{mn} / d\ln t_{kl}$ is provided by $\frac{d\ln \tau_{mn}}{d\ln t_{kl}} = \left(\frac{\tau_{mn}}{\tau_{mk} t_{kl} \tau_{ln}} c \right)^{\theta}$.

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