Appendix B (not for publication)

6.1 The Probabilistic Voting Model

In this section of the online appendix, we reproduce, for the reader's convenience, the description of the probabilistic voting model, based on Lindbeck and Weibull (1987) and Persson and Tabellini (2000), and applied to a dynamic voting setting by SSZ12. This material is also covered in the online appendix of SSZ12, and does not represent an original contribution of this paper.

The population has a unit measure and consists of two groups of voters, young and old, of equal size (we discussed below the extension to groups of different sizes). The electoral competition takes place between two office-seeking candidates, denoted by A and B. Each candidate announces a fiscal policy vector, b', τ, and g, subject to the government budget constraint, $b' = Rb + g - w(R)\, \tau\, H(\tau)$, and to $b' \leq \bar{b}$.[25] Since there are new elections every period, the candidates cannot make credible promises over future policies (i.e., there is lack of commitment beyond the current period). Voters choose either of the candidates based on their fiscal policy announcements and on their relative *appeal*, where the notion of appeal is explained below. In particular, a young voter prefers candidate A over B if, given the inherited debt level b, preference parameter θ, the world interest rate, and the equilibrium policy functions $\langle B, G, T \rangle$ which apply from tomorrow and onwards,

$$U_Y\left(\tau_A, g_A, G\left(b'_A\right); b, \theta, R\right) > U_Y\left(\tau_B, g_B, G\left(b'_B\right); b, \theta, R\right).$$

Likewise, a young voter prefers candidate A over B if

$$U_O\left(g_A; b, \theta, R\right) > U_O\left(g_B; b, \theta, R\right).$$

σ^{iJ} (where $J \in \{Y, O\}$) is an individual-specific parameter drawn from a symmetric group-specific distribution that is assumed to be uniform in the support $[-1/\left(2\kappa^J\right), 1/\left(2\kappa^J\right)]$. Intuitively, a positive (negative) σ^{iJ} implies that voter i has a bias in favor of candidate B (candidate A). Note that the distributions have density κ^J and that neither group is on average biased towards either candidate. The parameter δ is an aggregate shock capturing the ex-post average success of candidate B whose realization becomes known after the policy platforms have been announced. δ is drawn from a uniform i.i.d. distribution on $[-1/(2\psi), 1/(2\psi)]$.[26] The sum of the terms $\sigma^{iJ} + \delta$ captures the relative appeal of candidate B: it is the inherent bias of individual i in group J for candidate B irrespective of the policy that the candidates propose. The assumption of uniform distributions is for simplicity (see Persson and Tabellini (2000), for a generalization).

Note that voters are rational and forward looking. They take into full account the effects of today's choice on future private and public-good consumption. Because of repeated elections, they cannot decide directly over future fiscal policy. However, they can affect it through their choice of next-period debt level (b'), which affects future policy choices through the equilibrium policy functions B, T, and G.

[25] Note that the announcement over the current fiscal policy raises no credibility issue, due to the assumption that the politicians are pure office seekers and have no independent preferences on fiscal policy.

[26] The realization of δ can be viewed as the outcome of the campaign strategies to boost the candidates' popularity. Such an outcome is unknown *ex ante*.

It is at this point useful to identify the "swing voter" of each group, i.e., the voter who is ex-post indifferent between the two candidates:

$$\begin{aligned}\sigma^Y\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right) &= U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)-\delta\\ \sigma^O\left(g_A,g_B;b,\theta,R\right) &= U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)-\delta.\end{aligned}$$

Conditional on δ, the vote share of candidate A is

$$\begin{aligned}&\pi_A\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\\ =\;& 1-\pi_B\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\\ =\;& \frac{1}{2}\kappa^Y\left(\sigma^Y\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)+\frac{1}{2\kappa^Y}\right)\\ &+\frac{1}{2}\kappa^O\left(\sigma^O\left(g_A,g_B;b,\theta,R\right)+\frac{1}{2\kappa^O}\right)\\ =\;& \frac{1}{2}+\frac{1}{2}\left(\kappa^Y\times\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)-\delta\right)\\ &+\frac{1}{2}\left(\kappa^O\times\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right)-\delta\right).\end{aligned}$$

Note that π_A and π_B are stochastic variables, since δ is stochastic. The probability that candidate A wins is then given by

$$\begin{aligned}p_A &= \text{Prob}_\delta\left[\pi_A\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\geq\frac{1}{2}\right]\\ &= \text{Prob}\left[\begin{array}{c}\delta<\frac{\kappa^Y}{\kappa^Y+\kappa^O}\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ +\frac{\kappa^O}{\kappa^Y+\kappa^O}\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right)\end{array}\right]\\ &= \frac{1}{2}+\psi\left(1-\omega\right)\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ &\quad+\psi\omega\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right),\end{aligned}$$

where $\omega\equiv\kappa^O/\left(\kappa^Y+\kappa^O\right)$.

Since both candidates seek to maximize the probability of winning the election, the Nash equilibrium is characterized by the following equations:

$$\begin{aligned}\left(b'^*_A,\tau^*_A,g^*_A\right) &= \max_{b'_A,\tau_A,g_A}\left(1-\omega\right)\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ &\quad+\omega\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right),\\ \left(b'^*_B,\tau^*_B,g^*_B\right) &= \max_{b'_B,\tau_B,g_B}\left(1-\omega\right)\left(U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)-U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)\right)\\ &\quad+\omega\left(U_O\left(g_B;b,\theta,R\right)-U_O\left(g_A;b,\theta,R\right)\right).\end{aligned}$$

Hence, the two candidates' platform converge in equilibrium to the same fiscal policy maximizing the weighted-average utility of the young and old,

$$\left(b'^*_A,\tau^*_A,g^*_A\right)=\left(b'^*_B,\tau^*_B,g^*_B\right)=\max_{b'\tau,g}\left(\left(1-\omega\right)U_Y\left(\tau,g,G\left(b'\right);b,\theta,R\right)+\omega U_O\left(g;b,\theta,R\right)\right),$$

subject to the government budget constraint. This is the recursive version of the planner's objective function, (1), given in the body of the paper.

Note that the parameter ω has a structural interpretation: it is a measure of the relative variability within the old group of the candidates' appeal. As shown above, κ^Y/κ^O (and, hence, ω) affects the number of swing voters in each group. For instance, suppose that $\kappa^O > \kappa^Y$. Intuitively, this means that the old are more "responsive" in electoral terms to fiscal policy announcements in favor of or against them. An alternative interpretation is that $1/\kappa^J$ measures the extent of group J heterogeneity with respect to other policy dimensions that are orthogonal to fiscal policy. For example, the young might work in different sectors and cast their votes also based on the sectoral policy proposed by each candidate. As a result, the vote of the young is less responsive to fiscal policy announcements, and the young have effectively less political power than the old. This interpretation is consistent with Mulligan and Sala-i-Martin (1999) and Hassler *et al.* (2005). In the extreme case of $\omega = 1$, the old only care about fiscal policy ($\kappa^O \to 0$) and the distribution has a mass point at $\sigma^O = 0$. In this case, the young have no influence and the old dictate their fiscal policy choice (as in the commitment solution with $\alpha = 0$).

Suppose, next, that the groups have different relative size, and that there are N_O old voters and N_Y young voters. Proceeding as above, the planner's objective function is then modified to

$$\begin{aligned} \left(b_A'^*, \tau_A^*, g_A^*\right) &= \left(b_B'^*, \tau_B^*, g_B^*\right) = \\ &= \max_{b'\tau, g} \left\{(1-\omega) N_Y U_Y \left(\tau, g, G\left(b'\right); b, \theta, R\right) + \omega N_O U_O \left(g; b, \theta, R\right)\right\} \end{aligned}$$

We conclude by noting that the probabilistic voting outlined in this appendix applies equally to both static and dynamic models (under the assumption of Markov Perfect Equilibrium). The political model entails some important restrictions. First, agents only condition their voting strategy on the payoff-relevant state variable (here, debt). Second, the shock δ is i.i.d. over time – otherwise, the previous realization of δ becomes a state variable, complicating the analysis substantially. Third, although the assumption of uniform distributions can be relaxed, it is necessary to impose regularity conditions on the density function in order to ensure that the maximization problem is well behaved.

6.2 Statement and Proof of Lemma 7

Lemma 7 *The program (7) subject to (5) and (6) is a contraction mapping. Hence, a solution exists and is unique.*

Proof. Consider the intra-temporal FOC, (10), that is derived in the text. The condition solves

$$g = \Theta(\tau), \tag{53}$$

with $\Theta'(\cdot) < 0$. We can rewrite the government budget constraint as

$$b' - Rb = \Lambda(\tau) \equiv \Theta(\tau) - \tau w H(\tau),$$

where $\Lambda : [0, \bar{\tau}] \to [-\bar{\tau} w H(\bar{\tau}), \Theta(0)]$ is monotonic. Therefore, $\tau = \Lambda^{-1}(b' - Rb)$. Then, (7) can be rewritten as

$$V_O^{comm}(b) = \max_{b' \in [\underline{b}, \bar{b}]} \left\{\hat{v}\left(b' - Rb\right) + \beta\lambda V_O^{comm}\left(b'\right)\right\}, \tag{54}$$

where

$$\hat{v}\left(b' - Rb\right) \equiv (1+\lambda)\, u\left(\Theta\left(\Lambda^{-1}\left(b' - Rb\right)\right)\right) + \lambda\phi\left(A\left(\Lambda^{-1}\left(b' - Rb\right)\right)\right).$$

Since the function $\hat{v}$ is bounded and continuous, and $\beta\lambda < 1$, Theorem 4.6 in Stokey and Lucas (1989) establishes that (54) has a unique fixed point. ■

6.3 Statement and Proof of Lemma 8

Lemma 8 *Assume that $\lambda > 0$, and*

$$(1+\lambda)\left(\left(\left(\Lambda^{-1}\right)'\right)^2\left(u''\left(\Theta'\right)^2 + u'\left(\Theta''\right)\right) + u'\Theta'\left(\Lambda^{-1}\right)''\right)$$

$$+\lambda\left(\left(\left(\Lambda^{-1}\right)'\right)^2\left(\phi''\left(A'\right)^2 + \phi' A''\right) + \phi' A'\left(\Lambda^{-1}\right)''\right) < 0, \tag{55}$$

where Λ is defined in the proof of Lemma 7. Then, the unique MPPE of Lemma 7 is a DMPPE.

Proof. The proof is an application of Theorem 2.1 in Santos (1991).[27] The theorem states that the policy functions are differentiable if (i) the return function $\hat{v}$ is strictly concave and (ii) optimal paths are strictly interior. Consider the formulation of the problem used in the proof of Lemma 7. Standard differentiation shows that the function $\hat{v}$ is strictly concave if and only if assumption (55) holds.

We must show that the optimal paths are interior; i.e., that for any $b \in \left(\underline{b}, \bar{b}\right)$,

$$B(b) \in \left(\underline{b}, \bar{b}\right).$$

Since setting $B(b) = \bar{b}$ would imply zero public expenditure in the next period, then $\lambda > 0$ ensures that $B(b) < \bar{b}$. It remains to prove that $B(b) > \underline{b}$ for any $b \in \left(\underline{b}, \bar{b}\right)$. Suppose instead that there exists a $\hat{b} \in \left(\underline{b}, \bar{b}\right)$ such that $B\left(\hat{b}\right) = \underline{b}$. The Euler equation must then be (recall that $\hat{v}'' < 0$);

$$\hat{v}'\left(\underline{b} - R\hat{b}\right) \leq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right). \tag{56}$$

By concavity of $\hat{v}$, $\hat{b} > \underline{b}$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) < \hat{v}'\left(\underline{b} - R\hat{b}\right)$. Equation (56) then implies

$$\hat{v}'\left(\underline{b} - R\underline{b}\right) < \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right) \tag{57}$$

Thus, if the agent is constrained for some $\hat{b} > \underline{b}$, she must be constrained for $b = \underline{b}$. Hence, $B\left(\underline{b}\right) = \underline{b}$. However, $\beta\lambda R \leq 1$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) \geq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right)$, which contradicts (57). This concludes the proof. ■

[27] Santos, Manuel "Smoothness of the Policy Function in Discrete Time Economic Models," *Econometrica*, 59 (1991), 1365-1382.

6.4 Statement and Proof of Proposition 3

For convenience, we restate the Proposition 3 already contained in the text.

Proposition 7 *[RESTATEMENT OF Proposition 3] Let* $\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \rangle$ *denote equilibrium policies when* $\omega = 1$. *Assume that* $\bar{B}(b), \bar{G}(b), \bar{T}(b)$ *are continuously differentiable. If*

$$\left| \frac{u''(g)}{\beta\lambda R u''(g')} - \bar{G}'(b') \left(1 - \frac{\phi'(A(\tau))\, u''(g)\, wH(\tau)(1-e(\tau))/u'(g)}{\phi''(A(\tau))\, A'(\tau) + \phi'(A(\tau))\, e'(\tau)/(1-e(\tau))}\right)\right| > 1,$$

where $g = \bar{G}(b)$, $g' = \bar{G}(b')$, $\tau = \bar{T}(b)$ *and* $b' = \bar{B}(b)$. *Then, for* ω *close to unity, there exists a unique DMPPE.*

Proof. The strategy of the proof is based on Judd (2004). Let $\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \rangle$, denote the equilibrium policies when $\omega = 1$. Time subscripts will denote partial derivatives. We rewrite the equilibrium conditions (12), (16) and (15) in the following form:

$$\frac{u'(\bar{G}(b))}{u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)} = \beta\lambda R - \beta\lambda G'\left(\hat{B}(\bar{G}(b), b)\right)\frac{1-\omega}{\lambda(1+\lambda\omega)}, \tag{58}$$

$$\hat{B}(\bar{G}(b), b) = \bar{G}(b) + Rb - \hat{T}(\bar{G}(b)) \cdot w \cdot H\left(\hat{T}(\bar{G}(b))\right), \tag{59}$$

$$\phi'\left(A\left(\hat{T}(\bar{G}(b))\right)\right) = \frac{1+\omega\lambda}{1-\omega(1-\lambda)}\left(1 - e\left(\hat{T}(\bar{G}(b))\right)\right) u'(\bar{G}(b)). \tag{60}$$

Let $\varepsilon \equiv \frac{\beta(1-\omega)}{\lambda(1+\lambda\omega)}$ where $\lim_{\omega\to 1}\varepsilon = 0$. We prove that in the neighborhood of $\varepsilon = 0$ there exists a unique policy function $G(b,\varepsilon)$ that solves the GEE, (58). Note that $G(b,\varepsilon)$ involves some slight abuse of notation. We plug-in the candidate equilibrium function $G(b,\varepsilon)$ into (58), obtaining

$$\Pi(\varepsilon, G(b,\varepsilon)) \equiv \frac{u'(G(b,\varepsilon))}{u'\left(G\left(\hat{B}(G(b,\varepsilon), b), \varepsilon\right)\right)} - \beta\lambda R + \varepsilon G_1\left(\hat{B}(G(b,\varepsilon), b), \varepsilon\right) = 0, \tag{61}$$

where we define

$$\hat{B}(G(b,\varepsilon), b) = G(b,\varepsilon) + Rb - \hat{T}(G(b,\varepsilon))\, H\left(\hat{T}(G(b,\varepsilon))\right). \tag{62}$$

Next, we differentiate (61) with respect to ε, and evaluate the resulting expression at $\varepsilon = 0$ (recalling that $G(b,0) = \bar{G}(b)$ and $G_1(b,0) = \bar{G}'(b)$).

$$\frac{u''(\bar{G}(b))\, G_2(b,0)}{u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)}$$
$$-\frac{u'(\bar{G}(b))\, u''\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)}{\left(u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)\right)^2}\left(\begin{array}{c}\bar{G}'\left(\hat{B}(\bar{G}(b), b)\right)\hat{B}_1(\bar{G}(b), b)\, G_2(b,0) \\ +G_2\left(\hat{B}(\bar{G}(b), b), 0\right)\end{array}\right)$$
$$+\bar{G}'\left(\hat{B}(\bar{G}(b), b)\right) = 0$$

After rearranging terms and using the fact that $u'\left(\bar{G}(b)\right) = \beta\lambda R u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)$ as implied by the GEE when $\varepsilon = 0$, we obtain:

$$\begin{aligned}
&-\frac{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) \\
&+\left(\begin{array}{c} -\frac{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)} \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \hat{B}_1\left(\bar{G}(b), b\right) \\ +\frac{u''\left(\bar{G}(b)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)} \end{array}\right) G_2(b, 0) \\
&+\bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) = 0
\end{aligned} \tag{63}$$

Therefore, (63) implies that:

$$G_2(b, 0) = J(b) \cdot G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) + Z(b), \tag{64}$$

where

$$\begin{aligned}
J(b) &= \left(\frac{u''\left(\bar{G}(b)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} - \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \hat{B}_1\left(\bar{G}(b), b\right)\right)^{-1} \\
Z(b) &= J(b) \cdot \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right)\left(-\frac{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}\right)
\end{aligned}$$

Note that (64) has an iterative nature. Define the mapping:

$$\left(\Upsilon\ G_2(b, 0)\right)(b) \equiv J(b) \cdot G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) + Z(b).$$

If $|J(b)| < 1$, then Υ is a contraction mapping. We now show $|J(b)| < 1$ if and only if assumption (17) holds. Differentiating equation (60), we solve

$$\hat{T}'\left(\bar{G}(b)\right) = \frac{\frac{1+\omega\lambda}{1-\omega(1-\lambda)}\left(1 - e\left(\bar{T}(b)\right)\right) u''\left(\bar{G}(b)\right)}{\phi''\left(A\left(\bar{T}(b)\right)\right) A'\left(\bar{T}(b)\right) + \frac{1+\omega\lambda}{1-\omega(1-\lambda)} e'\left(\bar{T}(b)\right) u'\left(\bar{G}(b)\right)}. \tag{65}$$

Differentiating equation (59), together with (65), leads to

$$\begin{aligned}
\hat{B}_1(G(b), b) &= 1 - \hat{T}'\left(\bar{G}(b)\right) wH\left(\bar{T}(b)\right)\left(1 - e\left(\bar{T}(b)\right)\right) \\
&= 1 - \frac{\frac{1+\omega\lambda}{1-\omega(1-\lambda)}\left(1 - e\left(\bar{T}(b)\right)\right) u''\left(\bar{G}(b)\right) wH\left(\bar{T}(b)\right)\left(1 - e\left(\bar{T}(b)\right)\right)}{\phi''\left(A\left(\bar{T}(b)\right)\right) A'\left(\bar{T}(b)\right) + \frac{1+\omega\lambda}{1-\omega(1-\lambda)} e'\left(\bar{T}(b)\right) u'\left(\bar{G}(b)\right)}.
\end{aligned}$$

Hence

$$|J(b)| = \left|\left(\frac{u''\left(\bar{G}(b)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} - \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \cdot \hat{B}_1\left(\bar{G}(b), b\right)\right)^{-1}\right| < 1,$$

by assumption (17) and the intra-temporal FOC (15) when $\omega = 1$. This establishes that Υ is a contraction mapping. Therefore, in the neighbuorhood of $\omega = 1$, there exists a unique derivative $G_2(b, 0)$.

Finally, we must show that the existence of a unique derivative $G_2(b, 0)$ establishes the existence of a unique equilibrium policy function, $G(b, \varepsilon)$, that satisfies the GEE. Differentiating the functional equation (61) with respect to ε and evaluating the result at $\varepsilon = 0$ lead to the linear operator equation

$$\Pi_1(0, G(b, 0)) + \Pi_2(0, G(b, 0)) G_2(b, 0) = 0.$$

The existence and the uniqueness of $G_2(b, 0)$ imply that $\Pi_2(0, G(b, 0))$ is invertible at neighborhood of $\varepsilon = 0$. Therefore, we can apply implicit function theorem (Judd, 2004, pp. 10) to show that there are neighborhoods ε_0 of $\varepsilon = 0$ and for all $\varepsilon \in \varepsilon_0$, there is a unique $G(b, \varepsilon)$. ■

6.5 Measurement of Public Goods

Our empirical measure of public good provision in the U.S. (from footnote 12) encompasses the following expenditure items: defense, highways, libraries, hospitals, health, employment security administration, veterans' services, air transportation, water transport and terminals, parking facilities, transit subsidies, police protection, fire protection, correction, protective inspection and regulation, sewerage, natural resources, parks and recreation, housing and community development, solid waste management, financial administration, judicial and legal, general public buildings, other government administration, and other general expenditures, not elsewhere classified.

Consumption is total personal consumption expenditures. The data source is the Economic Report of the President, tables B1, B20, and B86.

6.6 Statement and Proof of Proposition 8

Claim 1 *The DMPPE is defined by the program:*

$$\langle G(s_{-1}, b), T(s_{-1}, b)\rangle = \arg\max_{\tau, g} \quad \tilde{\nu}(\tau, g, s; s_{-1}) + \left(\tilde{\delta} - 1\right)\lambda u(A(\tau) - s, g) + \tilde{\delta}\beta\lambda V_O(s, b'),$$

where $\tilde{\nu}(\tau, g, s; s_{-1}) = u(Rs_{-1}, g) + \lambda u(A(\tau) - s, g)$, $\tilde{\delta} = 1 + (1 - \omega)/(\lambda\omega)$, $g' = G(s, b')$, $s = \tilde{S}(\tau, g, g')$, *and* $b' = Rb + g - \tau w H(\tau)$. V_O *is a fixed point of the following functional equation:*

$$V_O(s_{-1}, b) = \tilde{\nu}(T(s_{-1}, b), G(s_{-1}, b), S(s_{-1}, b); s_{-1}) + \beta\lambda V_O(S(s_{-1}, b), B(s_{-1}, b)),$$

where S *satisfies the equation:*

$$S(s_{-1}, b) = \tilde{S}(T(s_{-1}, b), G(s_{-1}, b), G(S(s_{-1}, b), B(s_{-1}, b))).$$

Proof. We write the planner's objective function in a sequential formulation.

$$\frac{\lambda}{1 - \omega + \omega\lambda} U = \frac{\omega\lambda}{1 - \omega + \omega\lambda} u(Rs_{-1}, g_0) + \lambda U_Y(\boldsymbol{s}, \boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$$

$$= \frac{\omega\lambda}{1-\omega+\omega\lambda} u(Rs_{-1}, g_0) + \lambda \sum_{t=0}^{\infty} (\lambda\beta)^t \left(u(A(\tau_t) - s_t, g_t) + \beta u(Rs_t, g_{t+1}) \right)$$

$$\begin{aligned} = & \frac{\omega\lambda}{1-\omega+\omega\lambda} u(Rs_{-1}, g_0) + \lambda u(A(\tau_0) - s_0, g_0) \\ & + \sum_{t=1}^{\infty} (\lambda\beta)^t \left(u(Rs_{t-1}, g_t) + \lambda u(A(\tau_t) - s_t, g_t) \right). \end{aligned}$$

Multiplying both sides by $\frac{1-\omega+\omega\lambda}{\lambda\omega}$ yields

$$= \tilde{\nu}(\tau_0, g_0, s_0; s_{-1}) + \left(\tilde{\delta} - 1\right) \lambda u(A(\tau_0) - s_0, g_0) + \tilde{\delta} \sum_{t=1}^{\infty} (\lambda\beta)^t \cdot \tilde{\nu}(\tau_t, g_t, s_t; s_{t-1})$$

$$= \tilde{\nu}(\tau_0, g_0, s_0; s_{-1}) + \left(\tilde{\delta} - 1\right) \lambda u(A(\tau_0) - s_0, g_0) + \tilde{\delta}\lambda\beta \cdot V_O(s_0, b_1),$$

where

$$\begin{aligned} V_O(s_0, b_1) &= \sum_{t=1}^{\infty} (\lambda\beta)^t \tilde{\nu}(\tau_t, g_t, s_t; s_{t-1}) \\ &= \sum_{t=1}^{\infty} (\lambda\beta)^t \tilde{\nu}(T(s_{t-1}, b_t), G(s_{t-1}, b_t), s_t; s_{t-1}), \end{aligned}$$

$s_t = S(s_{t-1}, b_t)$ and $b_{t+1} = B(s_{t-1}, b_t)$. The second equality follows from the fact that future variables must follow the equilibrium policy rules. Note in particular that in equilibrium, saving satisfies

$$S(s_{-1}, b) = \tilde{S}(T(s_{-1}, b), G(s_{-1}, b), G(S(s_{-1}, b), B(s_{-1}, b))).$$

Since $\beta\lambda < 1$, V_O can be expressed recursively as

$$V_O(s_{-1}, b) = \tilde{\nu}(T(s_{-1}, b), G(s_{-1}, b), S(s_{-1}, b); s_{-1}) + \beta\lambda V_O(S(s_{-1}, b), B(s_{-1}, b)).$$

This concludes the proof of the preliminaries. ■

Proposition 8 *Let $u = u(c, g)$, where $u_c > 0$, $u_g > 0$, and u is a quasi-concave function. Then, a DMPPE is characterized by a system of two functional equations:*

1. *A trade-off between private and public good consumption*

$$\begin{aligned} \tilde{\delta}\lambda u_1(c_Y, g) A'(\tau) = & \left(u_2(Rs_{-1}, g) + \tilde{\delta}\lambda u_2(c_Y, g) \right) \\ & \cdot (1 - e(\tau)) A'(\tau) \\ & + \beta\lambda \left(\tilde{\delta} - 1\right) \frac{u_2(Rs, g') G_1(s, b')}{1 - \tilde{S}_3(\tau, g, g') G_1(s, b')} \\ & \cdot \left(\tilde{S}_2(\tau, g, g') (1 - e(\tau)) A'(\tau) - \tilde{S}_1(\tau, g, g') \right). \end{aligned} \tag{66}$$

where subscripts denote partial derivatives, and the following equilibrium conditions hold

$$\begin{aligned} c_Y &= A(\tau) - s,\ c'_Y = A(\tau') - s', \\ c'_O &= Rs,\ c''_O = Rs', \\ g &= G(s_{-1}, b),\ g' = G(s, b'),\ g'' = G(s', b'') \\ \tau &= T(s_{-1}, b),\ \tau' = T(s, b'), \\ s &= \tilde{S}(\tau, g, g'),\ s' = \tilde{S}(\tau', g', g''), \\ b' &= g + Rb - \tau w H(\tau) \equiv B(s_{-1}, b),\ b'' = B(s, b'). \end{aligned}$$

2. *A Generalized Euler Equation (GEE) for public good consumption:*

$$\begin{aligned} &\frac{u_2(Rs_{-1}, g) + \tilde{\delta}\lambda u_2(c_Y, g)}{u_2(Rs, g')} \\ &+\beta\lambda\left(\tilde{\delta} - 1\right)\left(\frac{G_2(s, b') + G_1(s, b')\tilde{S}_2(\tau, g, g')}{1 - \tilde{S}_3(\tau, g, g')G_1(s, b')}\right) \\ = \ & R\beta\lambda\left(\begin{array}{c} 1 + \tilde{\delta}\lambda\frac{u_2(c'_Y, g')}{u_2(c'_O, g')} \\ +\beta\lambda\left(\tilde{\delta} - 1\right)\frac{u_2(c''_O, g'')}{u_2(c'_O, g')}G_1(s', b'')\frac{\tilde{S}_2(\tau', g', g'') + \tilde{S}_3(\tau', g', g'')G_2(s', b'')}{1 - \tilde{S}_3(\tau', g', g'')G_1(s', b'')} \end{array}\right). \end{aligned} \tag{67}$$

where c_Y, c'_Y, c'_O, c''_O, g, g', g'', τ, τ', s, s', b' and b'' are equilibrium values defined as above.

Proof. We start the proof from an analysis of the effect of τ and g on private savings. Taking the total differential of the saving function, $s = \tilde{S}(\tau, g, g') = \tilde{S}(\tau, g, G(s, b'))$, with respect to τ and g yields, respectively,

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}\tau} &= \tilde{S}_1(\tau, g, g') + \tilde{S}_3(\tau, g, g')\left(G_1(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - G_2(s, b')(1 - e(\tau))wH(\tau)\right) \Rightarrow \\ \frac{\mathrm{d}s}{\mathrm{d}\tau} &= \frac{\tilde{S}_1(\tau, g, g') - \tilde{S}_3(\tau, g, g')G_2(s, b')(1 - e(\tau))wH(\tau)}{1 - \tilde{S}_3(\tau, g, g')G_1(s, b')}, \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}g} &= \tilde{S}_2(\tau, g, g') + \tilde{S}_3(\tau, g, g')\left(G_1(s, b')\frac{\mathrm{d}s}{\mathrm{d}g} + G_2(s, b')\right) \Rightarrow \\ \frac{\mathrm{d}s}{\mathrm{d}g} &= \frac{\tilde{S}_2(\tau, g, g') + \tilde{S}_3(\tau, g, g')G_2(s, b')}{1 - \tilde{S}_3(\tau, g, g')G_1(s, b')}, \end{aligned}$$

where we note that

$$\frac{\frac{\mathrm{d}s}{\mathrm{d}\tau}}{(1 - e(\tau))wH(\tau)} + \frac{\mathrm{d}s}{\mathrm{d}g} = \frac{\frac{\tilde{S}_1(\tau, g, g')}{(1 - e(\tau))wH(\tau)} + \tilde{S}_2(\tau, g, g')}{1 - \tilde{S}_3(\tau, g, g')G_1(s, b')}. \tag{68}$$

Now consider the problem defined in Claim 1. The FOC w.r.t. τ is:

$$0 = \tilde{\delta}\lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) + \tilde{\delta}\beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - \tilde{\delta}\beta\lambda V_{O2}(s, b')(1 - e(\tau))wH(\tau). \tag{69}$$

The FOC w.r.t. g is

$$0=\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)+\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\tilde{\delta}\beta\lambda V_{O2}\left(s,b'\right). \tag{70}$$

We first derive (66), and then derive (67).

Derivation of (66). We claim (proof below):

$$V_{O1}\left(s_{-1},b\right)=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right). \tag{71}$$

Next, we combine (69) and (70) to substitute out $V_{O2}\left(s,b'\right)$:

$$\begin{aligned}
&\frac{\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-A'\left(\tau\right)+\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)-\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}\tau}}{\left(1-e\left(\tau\right)\right)wH\left(\tau\right)}\\
=\ &\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)\\
&+\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
\tilde{\delta}\lambda u_1\left(c_Y,g\right)A'\left(\tau\right)\ =\ &\Big(u_2\left(Rs_{-1},g\right)+\tilde{\delta}\lambda u_2\left(c_Y,g\right)\Big)\\
&\cdot\left(1-e\left(\tau\right)\right)A'\left(\tau\right)\\
&+\beta\lambda\left(\tilde{\delta}-1\right)\frac{u_2\left(Rs,g'\right)G_1\left(s,b'\right)}{1-\tilde{S}_3\left(\tau,g,g'\right)G_1\left(s,b'\right)}\\
&\cdot\Big(\tilde{S}_2\left(\tau,g,g'\right)\left(1-e\left(\tau\right)\right)A'\left(\tau\right)-\tilde{S}_1\left(\tau,g,g'\right)\Big).
\end{aligned}$$

where the first step uses (71) and the second step follows from equation (68) and from the household Euler equation, $u_1\left(A\left(\tau\right)-s,g\right)=\beta Ru_1\left(Rs,g'\right)$. The last expression is the "trade-off between private and public good consumption", (66).

Derivation of (67). We claim (proof below):

$$V_{O2}\left(s_{-1},b\right)=\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_2\left(s_{-1},b\right)+\beta\lambda RV_{O2}\left(s,b'\right). \tag{72}$$

Next, we use (71) to substitute out $V_{O1}\left(s,b'\right)$ in use (70), and use the household Euler equation, $u_1\left(A\left(\tau\right)-s,g\right)=\beta Ru_1\left(Rs,g'\right)$, to simplify terms. This yields:

$$\begin{aligned}
0\ =\ &\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)\\
&+\beta\lambda\left(\tilde{\delta}-1\right)u_2\left(Rs,g'\right)G_1\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\tilde{\delta}\beta\lambda V_{O2}\left(s,b'\right).
\end{aligned}$$

(72) implies that

$$
\begin{aligned}
& \frac{\tilde{\delta}\lambda u_2\left(c_Y,g\right)+u_2\left(Rs_{-1},g\right)}{u_2\left(Rs,g'\right)} \\
& +\beta\lambda\left(\tilde{\delta}-1\right)\left(\frac{G_2\left(s,b'\right)+G_1\left(s,b'\right)\tilde{S}_2\left(\tau,g,g'\right)}{1-\tilde{S}_3\left(\tau,g,g'\right)G_1\left(s,b'\right)}\right) \\
= \;& R\beta\lambda\left(\begin{array}{c} 1+\tilde{\delta}\lambda\frac{u_2\left(c_Y',g'\right)}{u_2\left(c_O',g'\right)} \\ +\beta\lambda\left(\tilde{\delta}-1\right)\frac{u_2\left(c_O'',g''\right)}{u_2\left(c_O',g'\right)}G_1\left(s',b''\right)\frac{\tilde{S}_2\left(\tau',g',g''\right)+\tilde{S}_3\left(\tau',g',g''\right)G_2\left(s',b''\right)}{1-\tilde{S}_3\left(\tau',g',g''\right)G_1\left(s',b''\right)}\end{array}\right).
\end{aligned}
$$

This expression is the GEE for public good consumption, (66). ■

6.6.1 Derivation of equations (71) and (72)

Claim 2 *The partial derivatives $V_{O1}\left(s_{-1},b\right)$ and $V_{O2}\left(s_{-1},b\right)$ can be expressed as:*

$$V_{O1}\left(s_{-1},b\right)=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right).$$

$$V_{O2}\left(s_{-1},b\right)=\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_2\left(s_{-1},b\right)+\beta\lambda RV_{O2}\left(s,b'\right).$$

Proof. Differentiating $V_O\left(s_{-1},b\right)$ w.r.t. s_{-1} yields

$$
\begin{aligned}
V_{O1}\left(s_{-1},b\right) \;=\;& \lambda u_1\left(A\left(\tau\right)-s,g\right)\left(A'\left(\tau\right)-\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)T_1\left(s_{-1},b\right) \\
& +\lambda\left(u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+u_2\left(A\left(\tau\right)-s,g\right)\right)G_1\left(s_{-1},b\right) \\
& +u_1\left(Rs_{-1},g\right)R+u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right) \\
& +\beta\lambda V_{O1}\left(s,b'\right)\left(\frac{\mathrm{d}s}{\mathrm{d}\tau}T_1\left(s_{-1},b\right)+\frac{\mathrm{d}s}{\mathrm{d}g}G_1\left(s_{-1},b\right)\right) \\
& +\beta\lambda V_{O2}\left(s,b'\right)\left(G_1\left(s_{-1},b\right)-\left(1-e\left(\tau\right)\right)wH\left(\tau\right)T_1\left(s_{-1},b\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
=\;& \left(\begin{array}{c}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(A'\left(\tau\right)-\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)+\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}\tau} \\ -\beta\lambda V_{O2}\left(s,b'\right)\left(1-e\left(\tau\right)\right)wH\left(\tau\right)\end{array}\right)T_1\left(s_{-1},b\right) \\
& +u_1\left(Rs_{-1},g\right)R+u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right) \\
& +\left(\begin{array}{c}-\lambda u_1\left(A\left(\tau\right)-s,g\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\lambda u_2\left(A\left(\tau\right)-s,g\right) \\ +\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\beta\lambda V_{O2}\left(s,b'\right)\end{array}\right)G_1\left(s_{-1},b\right).
\end{aligned}
$$

$$=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right),$$

which is equation (71).
Similarly, differentiating $V_O(s_{-1}, b)$ w.r.t. b yields

$$\begin{aligned}
V_{O2}(s_{-1}, b) &= \lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) T_2(s_{-1}, b) \\
&\quad + \lambda\left(u_1(A(\tau) - s, g)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right) + u_2(A(\tau) - s, g)\right) G_2(s_{-1}, b) \\
&\quad + u_2(Rs_{-1}, g) G_2(s_{-1}, b) \\
&\quad + \beta\lambda V_{O1}(s, b')\left(\frac{\mathrm{d}s}{\mathrm{d}\tau} T_2(s_{-1}, b) + \frac{\mathrm{d}s}{\mathrm{d}g} G_2(s_{-1}, b)\right) \\
&\quad + \beta\lambda V_{O2}(s, b')\left(-(1 - e(\tau)) wH(\tau) T_2(s_{-1}, b) + G_2(s_{-1}, b) + R\right)
\end{aligned}$$

$$\begin{aligned}
&= \begin{pmatrix} \lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) + \\ \beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - \beta\lambda V_{O2}(s, b')(1 - e(\tau)) wH(\tau) \end{pmatrix} T_2(s_{-1}, b) \\
&\quad + \beta\lambda R V_{O2}(s, b') \\
&\quad + \begin{pmatrix} \lambda u_1(A(\tau) - s, g)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right) + \lambda u_2(A(\tau) - s, g) \\ + \beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}g} + \beta\lambda V_{O2}(s, b') + u_2(Rs_{-1}, g) \end{pmatrix} G_2(s_{-1}, b)
\end{aligned}$$

$$= \left(1 - \frac{1}{\tilde{\delta}}\right) u_2(Rs_{-1}, g) G_2(s_{-1}, b) + \beta\lambda R V_{O2}(s, b'),$$

which is equation (72). ■