Liquidity Rules and Credit Booms*

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Abstract

This paper shows that liquidity regulation can trigger unintended credit booms in the presence of interbank market power. We consider a price-setter and a continuum of price-takers who trade reserves after the realization of idiosyncratic liquidity shocks. The price-takers are endogenously less liquid and circumvent regulation by engaging in shadow banking, which leads to a reallocation of funding away from the more liquid price-setter. This reallocation channel underlies the credit boom. Endogenous responses in bank liquidity ratios also affect the magnitude of the boom. We discuss extensions of the model and illustrate its quantitative performance with an application to China.

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1 Introduction

A decade after the 2007-09 financial crisis, the debate about bank regulation remains unsettled. Politicians cite the crisis as prima facie evidence of under-regulation, central bankers are weary of the unintended consequences of over-regulation, and the academic literature has yet to agree on what would constitute optimal regulation. In the words of Stanley Fischer, former Vice Chairman of the U.S. Federal Reserve, a "tightening in regulation of the banking sector may push activity to other areas – and things happen." Exactly what happens, he argues, is difficult to predict as there is limited theoretical work on the interactions between regulated and unregulated institutions and the economic incentives that drive them.[1]

These gaps in our understanding seem especially pronounced when it comes to liquidity regulation. Diamond and Kashyap (2016) characterize the post-crisis liquidity rules agreed upon by the Basel Committee on Bank Supervision as "a situation where practice is ahead of both theory and measurement." Allen and Gale (2017) go even further in their survey of existing literature and conclude that "with liquidity regulation, we do not even know what to argue about." Understanding which features of the economic environment are important for shaping the aggregate effects of liquidity regulation would thus propel the literature forward. In this paper, we establish that interbank market power is, to first order, one such feature.

Our model is one where banks engage in maturity transformation, borrowing short and lending long in the spirit of Diamond and Dybvig (1983). Maturity transformation leaves banks vulnerable to idiosyncratic withdrawal shocks, giving rise to an ex post interbank market where banks with insufficient liquidity (i.e., reserves) can borrow from banks with surplus liquidity at an endogenously determined price. Such interbank markets exist in Bhattacharya and Gale (1987) and Allen and Gale (2004). We then add two ingredients to this environment.

The first ingredient is that banks differ in their ability to set prices on the interbank

[1] Speech delivered at the 2015 Financial Stability Conference, Washington D.C., December 3, www.federalreserve.gov/newsevents/speech/fischer20151203a.htm.

market. We model a price-setting bank and a continuum of individually small price-taking banks. The price-setting bank is large in the sense of being non-atomistic. It internalizes that its demand for liquidity will increase the price of liquidity on the interbank market. Thus, the price-setter chooses to be more liquid than the price-takers, as captured by a lower ratio of long-term lending to short-term borrowing. The small banks, as interbank price-takers, would then be endogenously more constrained by the introduction of a liquidity regulation that caps the ratio of long-term lending to short-term borrowing at each bank.

The second ingredient is that each bank can choose how much maturity transformation to conduct in the regulated sector and how much to conduct outside the perimeter of regulation. We show that, in response to liquidity regulation, the interbank price-takers ("the small banks") find it optimal to offer a new savings instrument and manage the funds raised by this instrument on a balance sheet that is not subject to the regulation (e.g., the funds are managed in an off-balance-sheet vehicle that can make the loans the bank cannot make on its balance sheet without violating the liquidity floor). This constitutes shadow banking: it achieves the same type of credit intermediation as a regular bank without appearing on a regulated balance sheet. It also achieves the same type of maturity transformation as a regular bank, with long-term assets financed by short-term liabilities.

To attract funds into off-balance-sheet instruments, the small banks offer their depositors interest rates in excess of the interest rate on traditional deposits. On the margin, the premium that a small bank is willing to pay for off-balance-sheet funding is exactly equal to the tax implicitly imposed on its deposits by a binding liquidity floor. All else constant, the emergence of a savings instrument that pays a premium relative to traditional deposits poaches some deposits away from other banks, namely the interbank price-setter. We show that the price-setter does not find it optimal to completely undo the reallocation of savings towards the small banks by offering equally high returns. Credit to the real economy then increases because the small banks, as interbank price-takers, make more long-term loans per unit of funding than the price-setter.

This reallocation channel serves as the basis for an unintended credit boom, where the post-regulation equilibrium is characterized by more credit per unit of savings than the pre-regulation equilibrium. Of course, changes in the equilibrium amount of credit also depend on changes in the liquidity ratios of the banks, not just on changes in the allocation of savings across them. Whether the reallocation channel is dominated in equilibrium by higher liquidity ratios depends on the initial allocation of funding. All else constant, the reallocation of savings towards the small banks increases the demand for liquidity on the interbank market in the state where these banks experience high withdrawals. This state will already have the highest liquidity demand if the initial funding share of the small banks is sufficiently large. The ex ante supply of liquidity would therefore have to increase to meet the increase in demand, with the adjustment coming from the equilibrium liquidity ratios. If instead the price-setter has a sufficiently high initial funding share, then the reallocation of savings decreases the maximum demand for liquidity across states and the credit boom prevails as an equilibrium outcome.

The result that an aggregate credit boom can be born from the introduction of a liquidity floor is surprising. However, our paper generates it by adding only two ingredients to an otherwise standard banking model: accounting standards that do not outlaw off-balance-sheet business and an interbank market that is not competitive. It is the combination of interbank market power and shadow banking that is problematic, not one rather than the other. In an extension with limited liability and a small probability of socially costly financial crises, we establish the existence of a simple liquidity floor that implements constrained efficiency if (i) the shadow banking technology does not exist or (ii) interbank rates are determined in a competitive equilibrium. With both shadow banking and interbank market power, however, there may not exist a simple liquidity floor that implements the planner's solution, and such a floor may actually be welfare-reducing because the credit boom exacerbates liquidity shortages in the crisis state.

Our next contribution is to use the model to explore recent developments in China's economy. We choose China for the following reasons. Between 2007 and 2014, the ratio of debt to GDP in China exploded from 110% to 200%. The ratio of private credit to private savings, a more conservative gauge, also experienced a substantial 10 percentage point increase over the same period. This credit boom appears to have occurred on the heels of stricter liquidity regulation. Around 2008, Chinese regulators began enforcing an old but hitherto neglected loan-to-deposit cap which forbade banks from lending more than 75% of their deposits to non-financial borrowers. Our model predicts that some credit booms are unintentionally caused by liquidity regulation so we are interested to know whether liquidity regulation can account for at least part of the Chinese experience.

We first establish heterogeneity in interbank market power among China's commercial banks. We then calibrate the model to Chinese data. The calibrated version of our model shows that loan-to-deposit enforcement alone generates over half of the increase in China's aggregate credit-to-savings ratio between 2007 and 2014. The Chinese experience is characterized by two other important facts: an increase in the average interbank rate and a convergence in on-balance-sheet liquidity ratios among banks. We show that these facts cannot be generated alongside a credit boom in the absence of interbank market power.

The increase in the average interbank rate is a strategic response by the price-setter to the regulatory arbitrage activities of the price-takers. Intuitively, the incentive to hold liquid assets is higher when liquidity is expected to be expensive. Therefore, by raising the average interbank rate, the price-setter can incentivize the small banks to become more liquid, loosening their regulatory constraint and lowering the incentives for arbitrage. The small banks then behave less aggressively in their quest for off-balance-sheet business, which leads to less funding being poached from the price-setter. The price-setting bank is increasing the price of liquidity for itself should it need to borrow on the interbank market, but it does so in exchange for less encroachment on its funding share.

As the average interbank rate rises, enough additional liquidity is elicited from the price-takers that the price-setting bank can decrease its liquidity ratio in favor of longer-term assets. Accordingly, there is convergence in the on-balance-sheet liquidity ratios of the two types of banks, as the on-balance-sheet ratio of the small banks trivially increases to comply with the regulation. Quantitatively, we find that the increase in the on-balance-sheet ratio of the price-takers is almost entirely undone by the increase in their shadow banking activities. The credit boom then reflects a reallocation of funding from the more liquid price-setter to the less liquid price-takers as well as a strategic decrease in the liquidity ratio of the price-setter.

We then pursue a quantitative extension that allows for multiple shocks to the Chinese economy: shocks to liquidity regulation, shocks to loan demand stemming from the fiscal stimulus package announced by China's State Council in late 2008, and money supply shocks. We find that loan demand shocks and money supply shocks produce counterfactual correlations between key market-determined interest rates, specifically interbank interest rates and spreads on the high-return savings instruments offered by small versus large banks. Allowing for all three shocks simultaneously, the model matches a broad set of empirical moments very closely, while still assigning a dominant role to variation in loan-to-deposit rules.

The price-setter's influence over the interbank market is undermined by central bank intervention. To this point, a central bank that is sufficiently responsive to interbank rate fluctuations can decrease the magnitude of the credit boom triggered by liquidity regulation, assuming all other parameters are held constant. This implication of our model is important because there are several settings where an automatic offset by the central bank does not exist, opening the door for interbank market power by large banks. For example, central banks in countries with managed exchange rates, including China, are bounded in their ability to lean against fluctuations in the interbank market. The Federal Reserve's history also includes long periods where a short-term policy rate was not targeted, and this was certainly the case in the U.S. National Banking Era prior to the creation of the Fed. Frequent encounters with the zero lower bound over the past decade have also challenged the speed and precision with which central banks affect all of the short-term rates at which banks trade. Our model can be extended in a variety of ways to study the implications of liquidity regulation in different circumstances. We present several extensions in this paper.

1.1 Related Literature

The literature on bank regulation is concerned with unintended consequences. Within this literature, there are many papers on capital requirements, largely because capital regulation was used more widely than liquidity regulation before the 2007-09 financial crisis. In the models of Harris, Opp, and Opp (2014), Plantin (2015), and Huang (2018), higher capital requirements lead to shadow banking, with various channels through which financial stability is affected. See also Acharya, Schnabl, and Suarez (2013), Demyanyk and Loutskina (2016), and Buchak, Matvos, Piskorski, and Seru (2018) for empirical evidence of regulatory arbitrage in the context of capital-related regulations. In the model of Begenau (2020), higher capital requirements make deposits scarce, lowering overall bank funding costs by enough to increase lending. A related model that allows for both traditional and shadow banks is studied in Begenau and Landvoigt (2018). Naturally, the rise of alternative investment opportunities for savers involves some migration of funding away from the traditional sector. Such opportunities arise with the emergence of shadow banking but also in environments where banks compete with public firms for equity capital, e.g., Allen, Carletti, and Marquez (2015). Migration of funding away from the traditional sector as a result of shadow banking also occurs in our paper and interacts novelly with heterogeneity in interbank market power to produce the reallocation channel that underlies our credit boom.

The liquidity problems experienced during the crisis and the subsequent introduction of global liquidity standards are now shifting attention towards liquidity regulation. Allen and Gale (2017) provide an excellent survey of this literature. Other recent contributions include Gorton, Laarits, and Muir (2020) who examine arbitrage during the U.S. National Banking Era to evaluate the merits of liquidity coverage ratios; Van den Heuvel (2018) who compares the welfare costs of liquidity and capital requirements; Adrian and Boyarchenko

(2018) who present a dynamic model where liquidity requirements are preferable to capital requirements as a prudential policy tool; Banerjee and Mio (2018) who find no evidence that bank lending fell after the U.K. tightened liquidity regulation in 2010; Jin and Xiong (2018) who argue that macroprudential reserve requirements will unintentionally push banks to choose greater currency mismatch; Davis, Korenok, Lightle, and Prescott (2020) who use experimental methods to explore whether liquidity regulations will improve interbank trade in response to shocks; Robatto (2019) who studies the desirability of liquidity policies during financial crises when the price of near-money assets transmits pecuniary externalities; and Aldasoro and Faia (2016) who argue that increasing the liquidity coverage ratio from 60% to 100% for all banks does not reduce systemic risk in a network model calibrated to European data.[2] An earlier contribution to which our paper most closely relates is Farhi, Golosov, and Tsyvinski (2007, 2009) who theoretically analyze the effect of liquidity regulation on market interest rates in a broad set of specifications.

We contribute to the literature on bank regulation by introducing interbank market power into the study of liquidity rules. Since the early work of Keeley (1990), Neumark and Sharpe (1992) and others, there has been growing interest in market power in banking. Several recent contributions have focused on developing new models with quantitative applications. Drechsler, Savov, and Schnabl (2017) study how the market power of banks over depositors explains the transmission of U.S. monetary policy since the mid-1990s; Egan, Hortaçsu, and Matvos (2017) develop a structural model of the U.S. banking sector to study financial stability with imperfect competition in deposit markets; Corbae and D'Erasmo (2013, 2019) study the relationship between bank entry, exit, and risk-taking when large banks are first-movers in lending markets. The focus of our paper is on interbank market power and its implications for the effectiveness of liquidity regulation. The result is a theory of unintended credit booms that also allows for careful calibration.

[2] In Aldasoro and Faia (2016), banks become more liquid to comply with the regulation but the liquidity is not available for interbank trade, weakening risk-sharing between banks. In our model, shadow banking emerges to evade compliance and the banking system becomes less liquid.

Research on past financial crises demonstrates the empirical importance of understanding interbank markets. Mitchener and Richardson (2019) show how a pyramid structure in U.S. interbank deposits propagated shocks during the Great Depression; Gorton and Tallman (2016) show how cooperation among members of the New York Clearinghouse helped end pre-Fed banking panics; and Frydman, Hilt, and Zhou (2015) show how a lack of cooperation with and between New York's trust companies contributed to the Panic of 1907. Separately, the decentralized nature of interbank trade has been explored through the lens of search theory (e.g., Duffie, Gârleanu, and Pedersen (2005), Ashcraft and Duffie (2007), Afonso and Lagos (2015)), with effective market power determined by the ease with which suitable counterparties can be found, and also in alternating-offer bargaining games (e.g., Acharya, Gromb, and Yorulmazer (2012)). Our assumption of market power is consistent with frictions in forming trading relationships, although we do not model such frictions explicitly. Instead, we focus on liquidity regulation and how it can be endogenously undermined when large banks have pricing power over small banks on the interbank market.

The implications of size asymmetries without market power have recently been explored by Craig and von Peter (2014), who show that large banks emerge as intermediaries in interbank trade because of economies of scale and scope, and by Dávila and Walther (2020), who show that large banks choose more leveraged positions than small banks when they internalize effects on the government's bailout policy. The predictions of Dávila and Walther (2020) on leverage are consistent with large banks in the U.S. being more constrained by capital requirements than small banks. We show that large banks choose more liquid positions than small banks when they have pricing power on the interbank market, which is consistent with large banks in China being less constrained by liquidity requirements than small banks. See Hachem (2018) for further discussion, including a comparison between shadow banking in the U.S. and China. While there are certainly other reasons why a large bank might hold more liquid assets than a small one, interbank market power proves necessary to understand the totality of China's experience with liquidity regulation.

Finally, our quantitative application is related to a rapidly growing literature on China's financial system. See Hachem (2018) and Song and Xiong (2018) for surveys. Chen, Ren, and Zha (2018) argue that an additional form of shadow banking emerged in China as an unintended consequence of contractionary monetary policy. The analysis is based on a different shock and a different set of financial products but their main finding echoes one of several findings original to our paper: policy tightenings in China have been undermined by the shadow banking products they triggered.

The rest of our paper is organized as follows. Section 2 presents the benchmark model and characterizes the unregulated equilibrium. Section 3 introduces liquidity regulation and establishes the main analytical results using a perturbation argument. Section 4 sketches the motivation for regulation and discusses additional extensions. Section 5 applies the model to China, presenting the calibration results along with a structural estimation to evaluate the importance of various shocks. Section 6 concludes. All proofs are collected in Online Appendix A.

2 Benchmark Model

There are three periods, $t \in \{0, 1, 2\}$, and two types of risk neutral banks, $i \in \{j, k\}$. Within each type, there can be one granular bank or a measure-one continuum of identical atomistic banks. We refer to the representative bank in type i as bank i. Let x_i^0 denote the funding obtained by bank i at $t = 0$. We normalize $x_j^0 + x_k^0 = 1$, in which case x_i^0 also constitutes the funding share of type i. While types j and k can differ in size, that is, we need not restrict attention to $x_i^0 = \frac{1}{2}$, the key difference between the two types of banks will lie in their ability to set prices on an interbank market for liquidity, as will be described below.

At $t = 0$, each bank i allocates its funding between liquid and illiquid assets. Denote by $\lambda_i \in [0, 1]$ the fraction of bank i's funding allocated to liquid assets (reserves). We model the illiquid asset as a project that returns $g\,(1 - \lambda_i)\, x_i^0$ at $t = 2$. The rate of return on bank i's project is then $\frac{g(1-\lambda_i)}{1-\lambda_i}-1$, which is independent of bank size. The return per unit of funding, $g(\cdot)$, has the following general properties:

Assumption 1 *(Properties of return function). $g(0)=0$, $\frac{g(1-\lambda)}{1-\lambda}>1$ for all $\lambda\in[0,1)$, $g'(\cdot)>0$, $g'(1)>1$, $g''(\cdot)<0$, $g'''(\cdot)\geq 0$.*

Projects are long-term, meaning that they run from $t=0$ to $t=2$ without the possibility of liquidation at $t=1$. To introduce a tradeoff between investing in the long-term project and holding reserves, banks are subject to short-term liquidity shocks which must be paid off at $t=1$. The realization of liquidity shocks across banks depends on the aggregate state. The economy can be in one of two states. State $s\in\{A,B\}$ occurs with probability $\pi_s\in(0,1)$, in which case fraction $\theta_i^s\in(0,1)$ of bank i's funding is withdrawn at $t=1$. It is not known until $t=1$ which state is realized. Later, we will allow for the possibility of a crisis state C which occurs with probability $\varepsilon\equiv 1-\pi_A-\pi_B$ and involves a run on the entire banking system, i.e., $\theta_j^C=\theta_k^C=1$. For now, however, we focus on the two non-crisis states. To this end, define $\widetilde{\pi}\equiv\frac{\pi_A}{\pi_A+\pi_B}$. The expected value of the liquidity shock is the same for all banks,

$$\widetilde{\pi}\theta_j^A+(1-\widetilde{\pi})\,\theta_j^B=\widetilde{\pi}\theta_k^A+(1-\widetilde{\pi})\,\theta_k^B$$

where $\theta_j^A>\max\left\{\theta_k^A,\theta_j^B\right\}$ and $\theta_k^B>\max\left\{\theta_k^A,\theta_j^B\right\}$. In words, bank j experiences more withdrawal pressure in one state ("state A") while bank k experiences more withdrawal pressure in the other ("state B").

2.1 The Interbank Market

The maturity mismatch between long-term projects and liquidity shocks follows the tradition of Diamond and Dybvig (1983). It also introduces a role for reserves that can be used to pay realized shocks at $t=1$. If $\theta_i^s<\lambda_i$, then bank i has a reserve surplus at $t=1$. If $\theta_i^s>\lambda_i$, then bank i has a reserve shortage at $t=1$. An interbank market exists at $t=1$

to redistribute reserves across banks. A market in which banks can share risk and obtain liquidity also exists in Bhattacharya and Gale (1987) and Allen and Gale (2004).

The interbank interest rate in state $s \in \{A, B\}$ is denoted by r_s. Interbank lenders (borrowers) are banks with reserve surpluses (shortages) at $t = 1$. The aggregate feasibility condition for state $s \in \{A, B\}$ is

$$\left(\lambda_k - \theta_k^s\right) x_k^0 + \left(\lambda_j - \theta_j^s\right)\left(1 - x_k^0\right) \geq 0 \tag{1}$$

The left-hand side of (1) captures the net demand for liquidity in the interbank market at $t = 1$. Aggregate feasibility states that there cannot be a market-wide liquidity shortage in either (non-crisis) state. The total amount of funding invested in long-term projects rather than allocated to reserves, $1 - \lambda_k x_k^0 - \lambda_j \left(1 - x_k^0\right)$, constitutes total credit in this economy.

We are interested in distributions of liquidity shocks across banks and states such that each bank sometimes borrows in the interbank market and other times lends. With two representative banks and aggregate feasibility in the two non-crisis states, this means that k lends when j borrows and vice versa. Without loss of generality, we have defined A to be the state where k lends and B the state where k borrows. Our focus will therefore be on shock distributions $\left\{\theta_j^A, \theta_j^B, \theta_k^A, \theta_k^B; \widetilde{\pi}\right\}$ that support $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ and $\lambda_k \in \left(\theta_k^A, \theta_k^B\right)$ as equilibrium outcomes, and we will verify the existence of such distributions later on.

2.2 Optimization Problem of Interbank Price-Takers

We model bank j, the representative bank in type j, as an interbank price-taker. Specifically, type j is made up of a continuum of price-taking banks who all experience the liquidity shock θ_j^s. A price-taker can trade any amount of liquidity at $t = 1$ in the interbank market at the interest rate r_s if state $s \in \{A, B\}$ is realized. Banks in the continuum are atomistic so they take r_s as given when making decisions.

Given funding x_j^0 and interbank rates r_A and r_B, bank j chooses its liquidity ratio λ_j to maximize its expected profit at $t = 0$. Formally, this optimization problem is

$$\max_{\lambda_j} \left\{ \widetilde{\pi} \Upsilon^A (\lambda_j; r_A) + (1 - \widetilde{\pi}) \Upsilon^B (\lambda_j; r_B) \right\}$$

where $\Upsilon^s (\cdot)$ denotes the ex post profit per unit of funding, i.e.,

$$\Upsilon^s (\lambda_j; r_s) \equiv g (1 - \lambda_j) + \lambda_j - 1 + r_s \left(\lambda_j - \theta_j^s \right) \tag{2}$$

The first term in Eq. (2) is the return from the long-term project, the second term is the value of reserves, the third term represents the repayment of funding (θ_i^s at $t = 1$ and $1 - \theta_i^s$ at $t = 2$), and the fourth-term is the interest income (or expense if negative) from lending (borrowing) reserves on the interbank market.

The first order condition with respect to λ_j is

$$g' (1 - \lambda_j) = 1 + E (r) \tag{3}$$

where we have defined the expected interbank rate $E (r) \equiv \widetilde{\pi} r_A + (1 - \widetilde{\pi}) r_B$. The left-hand side of Eq. (3) is the marginal cost of increasing reserves, namely the marginal return from the long-term project, while the right-hand side is the marginal benefit, namely the expected return from lending a unit of reserves on the interbank market. The solution to Eq. (3) is the same across the continuum of price-taking banks.

2.3 Optimization Problem of Interbank Price-Setter

Bank k has market power in interbank trading, that is, k is a price-setter of r_A and r_B, not a price-taker. For simplicity, we assume one granular bank with interbank market power relative to the continuum of price-takers. In practice, one could imagine a finite number of price-setters, each having market power on a subset of the continuum because of frictions in forming trading relationships. A central bank that actively intervenes in the interbank market, i.e., adding/removing liquidity when the interbank rate increases/decreases, would change Eq. (1) and undermine the price-setting power of any bank k. We consider central bank liquidity interventions in an extension later in the paper.

Given a funding share x_k^0, bank k chooses its liquidity ratio λ_k and the interbank rates r_A and r_B to maximize its expected profit at $t = 0$,

$$\Upsilon_k \equiv [g(1-\lambda_k) + \lambda_k - 1] x_k^0 + \left[\widetilde{\pi} r_A \left(\theta_j^A - \lambda_j\right) + (1-\widetilde{\pi}) r_B \left(\theta_j^B - \lambda_j\right)\right] \left(1 - x_k^0\right) \quad (4)$$

subject to (i) the best response of λ_j to the expected interbank rate $E(r)$ in Eq. (3), (ii) aggregate feasibility as per Eq. (1) for each state $s \in \{A, B\}$, and (iii) $r_s \in [0, \overline{r}_s]$ for each state $s \in \{A, B\}$. We restrict attention to non-negative rates, $r_s \geq 0$, because reserves can be stored between $t = 1$ and $t = 2$ at rate of return zero instead of lent on the interbank market. We also introduce a ceiling $\overline{r}_s \geq 0$, which will be discussed further in Section 2.4.

Notice from Eq. (4) that j's net demand for liquidity, $\left(\theta_j^s - \lambda_j\right)\left(1 - x_k^0\right)$ in state $s \in \{A, B\}$, determines the size of k's interbank trades. This reflects the fact that bank k is a price-setter and therefore cannot trade whatever quantity of liquidity it would like at the prevailing interbank rate. The following lemma is then immediate from the properties of $g(\cdot)$ in Assumption 1, specifically $g'(1) > 1$ and $g''(\cdot) < 0$:

Lemma 1 *(Aggregate feasibility binds). Bank k will never choose a liquidity ratio λ_k that makes Eq. (1) slack in both states $s \in \{A, B\}$.*

A formal statement of the proof of Lemma 1 is collected into the proof of Proposition 1 below. Intuitively, k cannot benefit from lending excess liquidity in a slack market and is therefore better off investing those funds in the long-term project. The price-setter's problem then simplifies to choosing $r_A \in [0, \overline{r}_A]$ and $r_B \in [0, \overline{r}_B]$ to maximize Eq. (4), taking into account that λ_j solves Eq. (3) and λ_k solves

$$\left(\lambda_k - \theta_k^{s'}\right) x_k^0 + \left(\lambda_j - \theta_j^{s'}\right)\left(1 - x_k^0\right) = 0 \quad (5)$$

where s' denotes the (non-crisis) state with the highest aggregate withdrawal pressure,

$$s' \equiv \arg\max_{s \in \{A,B\}} \left\{\theta_j^s \left(1 - x_k^0\right) + \theta_k^s x_k^0\right\}$$

Clearly, $s' = B$ if and only if $x_k^0 > \left(1 + \frac{\theta_k^B - \theta_k^A}{\theta_j^A - \theta_j^B}\right)^{-1} \equiv \underline{x}_k^0$.

Our formulation of bank k's problem implicitly assumes commitment, i.e., k announces state-contingent interbank rates at $t = 0$ which are then honored at $t = 1$. We discuss the no commitment case in Online Appendix B as a robustness exercise.

2.4 Unregulated Equilibrium

We now formally define an equilibrium in the absence of regulation.

Definition 1 *An (unregulated) equilibrium given the funding shares x_k^0 and $x_j^0 \equiv 1 - x_k^0$ consists of liquidity ratios (λ_j, λ_k) for banks j and k and interbank interest rates (r_A, r_B) for states A and B such that each bank solves its optimization problem at $t = 0$ and aggregate feasibility holds at $t = 1$, that is, (i) λ_j satisfies the first order condition in Eq. (3) conditional on the expected interbank rate $E(r)$ and (ii) $r_A \in [0, \overline{r}_A]$ and $r_B \in [0, \overline{r}_B]$ maximize Eq. (4) subject to λ_j as per Eq. (3) and λ_k as per Eq. (5).*

Denote by $\{\lambda_j^*, \lambda_k^*, r_A^*, r_B^*\}$ the equilibrium values of the endogenous variables. The following proposition shows that the price-setting bank elects to be more liquid than the price-taking banks if there exists a state of the world where the price-setter borrows from the price-takers at positive interest rate.

Proposition 1 *(Cross-sectional differences in liquidity ratios). $\lambda_k^* > \lambda_j^*$ in any equilibrium where $\lambda_j^* \in \left(\theta_j^B, \theta_j^A\right)$ and $r_B^* > 0$.*

The intuition for this result is as follows. Bank k understands from Eq. (5) that decreasing the liquidity ratio λ_k will require an increase in the liquidity ratio of the price-takers λ_j to satisfy aggregate feasibility in state s'. To incentivize higher λ_j among the price-takers, k would have to increase $E(r)$ in Eq. (3). This increase can come from the interbank rate in either state, r_A or r_B. We notice from Eq. (5) that $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ implies $\lambda_k \in \left(\theta_k^A, \theta_k^B\right)$, hence k is an interbank borrower in state B in the equilibrium considered in Proposition 1.

If also $r_B > 0$, then k must have already set r_A as high as possible otherwise an arbitrarily small deviation $\Delta r_A > 0$, with $\Delta r_B = -\frac{\widetilde{\pi}}{1-\widetilde{\pi}} \Delta r_A < 0$ to keep $\Delta E(r) = 0$, would increase k's expected profit in Eq. (4) by $\widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)(1 - x_k^0)\,\Delta r_A > 0$. The equilibrium therefore has $r_A = \overline{r}_A$ if it has $r_B > 0$, in which case the increase in $E(r)$ to incentivize higher λ_j has to come from an increase in r_B. The price-setting bank internalizes this negative relationship between its liquidity ratio and its interbank borrowing costs, in contrast to the price-takers who choose liquidity ratios taking as given all interbank prices. Accordingly, $\lambda_k > \lambda_j$.

Next, we establish existence of an equilibrium with the properties considered in Proposition 1:

Proposition 2 *(Existence of equilibrium). There exists a unique equilibrium with* $\lambda_j^* \in \left(\theta_j^B, \theta_j^A\right)$ *and* $r_B^* > 0$ *if* θ_j^A *is sufficiently high and* $\overline{r}_A$ *is sufficiently low.*

Recall from Eq. (3) that j's choices only depend on the expected interbank rate $E(r)$. Thus, if k can set r_A as high as it wants without any ramifications, it will set $r_B = 0$ and use only r_A to influence $E(r)$ in any equilibrium where it lends in state A and borrows in state B. In other words, a price-setting bank will only pay positive interest in state B if it wants to incentivize higher λ_j but cannot extract more rents in state A. The condition on $\overline{r}_A$ in Proposition 2 is what delivers an equilibrium with $r_B > 0$. We consider $r_B > 0$ to be the empirically relevant equilibrium as a bank would have to possess an unrealistically high degree of market power to be able to borrow for free.

There are various interpretations of $\overline{r}_s$. One is an outside option (e.g., a state-contingent central bank discount window) that limits how much a bank would be willing to pay for funding on the interbank market. Another is limited liability on interbank trades, which would bound r_s by the solvency of the interbank borrower in state s. Online Appendix C presents this microfoundation. We show that the condition for ex post solvency of the price-takers in state A constrains the price-setter's choice of r_A by enough to deliver an equilibrium with the features in Proposition 1 when the liquidity shock θ_j^A is sufficiently high. Beyond this intuition, however, microfounding $\overline{r}_s$ as an ex post solvency constraint does not produce much additional insight so we keep $\overline{r}_s$ constant to simplify the exposition.[3]

3 Effect of Liquidity Regulation

We now introduce into the benchmark model a government-imposed liquidity floor on each bank, namely a requirement which says the ratio of reserves to funding must be at least $\alpha \in (0, 1)$. Given the structure of our model, reserves are meant to be used at $t = 1$ so enforcement of the liquidity rule is confined to $t = 0$. If the government does not enforce a liquidity rule, then $\alpha = 0$.

3.1 Shadow Banking

The liquidity rule, like all formal and enforceable bank regulation, only applies to activities that a bank reports on its balance sheet. To model this, we allow banks to choose where to manage the funding they receive. Specifically, by taking an action at cost ξ_i per unit of funding, bank i can divert a fraction $h(\xi_i)$ of its funding into an off-balance-sheet vehicle, away from the purview of regulation. Without loss of generality, all of the funding moved into off-balance-sheet vehicles is invested in long-term projects.[4]

Off-balance-sheet vehicles can be viewed as accounting maneuvers that legally shift activities away from regulation without changing the nature of those activities. Such maneuvers capitalize on the discretion available in accounting rules and constitute regulatory arbitrage,

[3] The key perturbation results in the next section are robust to the extension in Online Appendix C. The quantitative results in Section 5 are also robust to such extension (see the sensitivity analysis reported in Online Appendix J). Naturally, $\overline{r}_s$ constant can be set low enough to also respect ex post solvency in equilibrium.

[4] For banks constrained by the liquidity rule, this is the optimal action; it would be counter-productive to decrease the liquidity ratio that the regulator observes by booking reserves in an off-balance-sheet vehicle. For an unconstrained bank, any off-balance-sheet activity is for competitive purposes (more on this below), so the bank is indifferent about where reserves are held, conditional on remaining unconstrained.

or shadow banking.[5] The effective regulatory constraint is thus:

$$\lambda_i \geq \alpha\left(1 - h\left(\xi_i\right)\right) \tag{6}$$

where the shadow banking technology has the following general properties:

Assumption 2 *(Properties of shadow banking technology). $h(0) = 0$, $h(\xi_i) > 0$ for $\xi_i > 0$, $h'(\cdot) > 0$ with $h(\infty) \rightarrow 1$, $h''(\cdot) \leq 0$, $h'''(\cdot) \geq 0$.*

We interpret ξ_i as monetary incentives offered by bank i to entice savers to move some of their funds from regulated products (e.g., traditional deposits held on bank balance sheets) to unregulated products (e.g., deposit-like products that are not explicitly guaranteed by the bank and thus bookable in off-balance-sheet vehicles). It is reasonable to assume that traditional deposits have a higher convenience value to savers, in which case bank i cannot costlessly move funds into deposit-like products, i.e., $h(0) = 0$. We have defined ξ_i to be a cost per unit of funding, so with total funding x_i, the overall cost of shadow banking to bank i is $\xi_i x_i$. To ease the exposition, this cost is payable at $t = 2$.

In equilibrium, funding shares may also respond to differences in monetary incentives across banks. Formally, we consider

$$x_k \equiv x_k^0 + \delta_1\left(\xi_k - \overline{\xi_j}\right) \tag{7}$$

where $\overline{\xi_j}$ is the average shadow banking action taken by type j and the parameter $\delta_1 > 0$ governs the intensity of competition between j and k. The atomistic banks in j take as given $\overline{\xi_j}$ and ξ_k, and, in any symmetric equilibrium, $\overline{\xi_j}$ will be such that the profit-maximizing choice of ξ_j equals $\overline{\xi_j}$ for each bank in type j. Naturally, k does not take any of the variables in Eq. (7) as given because it is a granular bank. We sketch a simple microfoundation for Eq. (7), alongside the property $h'(\cdot) > 0$ as per Assumption 2, in Online Appendix D.

[5] Adrian, Ashcraft, and Cetorelli (2013) define regulatory arbitrage as "a change in structure of activity which does not change the risk profile of that activity, but increases the net cash flows to the sponsor by reducing the costs of regulation."

To simplify the analytical exposition, each bank in type j takes as given its equilibrium funding $x_j \equiv 1 - x_k$, i.e., these banks do not choose ξ_j with the intention of changing how much funding they receive. We relax this in an extension to the quantitative model.

3.2 Optimization Problems

Given funding x_j and interbank rates r_A and r_B, the representative bank j now chooses its liquidity ratio λ_j and its shadow banking action $\xi_j \geq 0$ to maximize its expected profit at $t = 0$,

$$\widetilde{\pi}\Upsilon^A(\lambda_j; r_A) + (1 - \widetilde{\pi})\Upsilon^B(\lambda_j; r_B) - \xi_j$$

subject to the liquidity floor in Eq. (6). The Lagrange multiplier on (6) is the shadow cost of holding reserves. We denote it by $\mu_j \geq 0$. The multiplier on $\xi_j \geq 0$ is denoted by $\rho_j \geq 0$. The first order conditions with respect to λ_j and ξ_j are then

$$g'(1 - \lambda_j) = 1 + E(r) + \mu_j \tag{8}$$

and

$$\rho_j = 1 - \alpha\mu_j h'(\xi_j) \tag{9}$$

respectively, with complementary slackness conditions

$$\mu_j\left[\lambda_j - \alpha\left(1 - h\left(\xi_j\right)\right)\right] = 0,\ \mu_j \geq 0,\ \lambda_j \geq \alpha\left(1 - h\left(\xi_j\right)\right) \tag{10}$$

$$\rho_j\xi_j = 0,\ \rho_j \geq 0,\ \xi_j \geq 0 \tag{11}$$

The optimal choice of λ_j in Eq. (8) still equates the marginal cost of increasing reserves with the marginal benefit. The difference relative to Eq. (3) is that reserves now also help to relax the constraint imposed by the liquidity floor, augmenting the marginal benefit by μ_j. The optimal choice of ξ_j in Eq. (9) can also be understood in terms of marginal costs and benefits. The marginal cost of the shadow banking action is 1, as ξ_j was defined to be the cost that j pays per unit of funding to divert fraction $h(\xi_j)$ away from regulation.

The marginal benefit is then $\alpha\mu_j h'(\xi_j)$. Notice that there is no marginal benefit to shadow banking if there is no regulation ($\alpha = 0$) or if the expected interbank rate is high enough to eliminate the shadow cost of holding reserves ($\mu_j = 0$ from Eqs. (8) and (10)). However, if $\alpha > 0$ and $\mu_j > 0$, bank j may have a regulatory arbitrage motive to engage in shadow banking, choosing $\xi_j > 0$ to solve Eq. (9) with $\rho_j = 0$.

The solution to Eqs. (8) to (11) is characterized in the following lemma:

Lemma 2 *(Best response of price-takers to expected interbank rate). Define*

$$\underline{R}(\alpha) \equiv g'(1-\alpha) - 1 - \frac{1}{\alpha h'(0)}$$

$$\overline{R}(\alpha) \equiv g'(1-\alpha) - 1$$

for a liquidity floor α. Bank j is constrained by regulation, as indicated by $\mu_j > 0$, if and only if $E(r) < \overline{R}(\alpha)$, and shadow banking $\xi_j > 0$ emerges if and only if $E(r) < \underline{R}(\alpha)$, where ξ_j is decreasing in $E(r)$.

The expected interbank rate $E(r)$ represents the expected cost of emergency liquidity at $t = 1$. When liquidity is expected to be very expensive, i.e., $E(r) > \overline{R}(\alpha)$, price-taking banks are incentivized to hold high liquidity ratios, irrespective of the liquidity floor. They are thus unconstrained by the floor and do not engage in shadow banking. When liquidity is expected to be moderately expensive, i.e., $E(r) \in (\underline{R}(\alpha), \overline{R}(\alpha))$, price-taking banks find it less profitable to hold reserves. They bump into the liquidity floor, but are not so constrained by it that they would profit from operating the shadow technology. Finally, when liquidity is expected to be fairly inexpensive, i.e., $E(r) < \underline{R}(\alpha)$, price-taking banks want to invest much more heavily in the long-term project to earn a better return, relying on the interbank market for cheap liquidity in the event of a high liquidity shock. They are thus constrained by the liquidity floor and engage in shadow banking to circumvent the regulation.

Consider now the problem of the price-setting bank. Bank k now chooses its liquidity ratio λ_k, the interbank rates $r_s \in [0, \overline{r}_s]$ for each state $s \in \{A, B\}$, and its shadow banking action $\xi_k \geq 0$ to maximize its expected profit at $t = 0$,

$$[g(1-\lambda_k) + \lambda_k - 1 - \xi_k]\, x_k + \left[\widetilde{\pi} r_A \left(\theta_j^A - \lambda_j\right) + (1-\widetilde{\pi})\, r_B \left(\theta_j^B - \lambda_j\right)\right](1 - x_k)$$

subject to (i) the best response of λ_j and ξ_j to $E(r)$ in Lemma 2, (ii) the determination of the funding share x_k in Eq. (7), (iii) aggregate feasibility as per Eq. (1) for each state $s \in \{A, B\}$, and (iv) the liquidity floor in Eq. (6).

From Lemma 2, λ_j and ξ_j only depend on r_A and r_B through $E(r)$, so by the same argument as in Section 2.4, bank k will set r_A as high as possible in any equilibrium where $r_B > 0$. The equilibrium must therefore have $r_A = \overline{r}_A$ if it has $r_B > 0$. We are interested in solutions with $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ and $r_B > 0$, so we work with $r_A = \overline{r}_A$ henceforth. This reduces k's choice variables to λ_k, ξ_k, and $E(r)$.

3.3 Equilibrium

We now study how the introduction of a liquidity floor α affects the equilibrium of the model.

Definition 2 *A (regulated) equilibrium consists of liquidity ratios (λ_j, λ_k) and shadow banking actions $\left(\xi_j, \xi_k\right)$ for banks j and k as well as interbank interest rates (r_A, r_B) for states A and B such that each bank solves its optimization problem at $t = 0$ (see Section 3.2), aggregate feasibility holds at $t = 1$, and funding shares are determined by Eq. (7).*

To fix ideas, consider $h'(0)$ arbitrarily large, i.e., $h'(0) \to \infty$, so that there is only one cutoff $\overline{R}(\alpha)$ in Lemma 2.[6] We demonstrate the main results using a perturbation argument. Specifically, we start in the unregulated equilibrium of Proposition 1, with $\lambda_k^* > \lambda_j^*$, then introduce a liquidity floor $\alpha = \lambda_j^*$ and analyze how the model responds to a slight increase in α.

All else constant, this perturbation to α will push $\overline{R}(\alpha)$ above the $E(r)$ in the unregulated equilibrium, which by Eq. (3) is equal to $\overline{R}\left(\lambda_j^*\right)$. Bank j will thus be constrained, prompting

[6] Another immediate implication of $h'(0) \to \infty$ is that the on-balance-sheet funding of the price-takers, $\left(1 - h\left(\xi_j\right)\right)(1 - x_k)$, decreases as ξ_j increases from zero, for any δ_1 finite in Eq. (7).

it to engage in the shadow banking activity, $\xi_j > 0$. Intuitively, the magnitude of ξ_j will reflect the magnitude of the Lagrange multiplier on the regulatory constraint. The tightening of liquidity rules effectively taxes on-balance-sheet activities relative to off-balance-sheet activities and, when the rule is tight enough to constrain banks, they are willing to pay to move funding off the balance sheet.

But all else is not constant. In particular, bank k could choose to increase $E(r)$ to some value $E(r) \geq \overline{R}(\alpha)$ so that the price-taking banks, as represented by bank j, are no longer constrained and therefore choose $\xi_j = 0$. We explore this next.

Lemma 3 *(Shadow banking by interbank price-takers). Fix $\xi_k = 0$. If the shadow banking technology has the properties $h'(0) \to \infty$ and $\frac{h''(0)}{(h'(0))^3} \to 0$, then (i) $\xi_j = 0$ for $\alpha = \lambda_j^*$ and (ii) $\xi_j > 0$ as α is perturbed above λ_j^*.*

Lemma 3 establishes that k will not find it optimal to increase $E(r)$ by as much as would be needed to keep $\xi_j = 0$ following an increase in α above λ_j^*. The condition $\frac{h''(0)}{(h'(0))^3} \to 0$ ensures that k also finds it suboptimal to lower $E(r)$ relative to the unregulated equilibrium once the regulation $\alpha = \lambda_j^*$ is introduced.[7]

We focus on the limiting case of $h'(0) \to \infty$ in the main text for ease of exposition. The case of $h'(0)$ large but not arbitrarily so is discussed in Online Appendix E. The local analysis yields qualitatively similar insights, except that shadow banking emerges around a

[7] It is worth elaborating briefly on this point. In any equilibrium with $r_B > 0$, bank k is setting $E(r)$ to incentivize the price-takers to share the burden of keeping the system liquid. In particular, we recall from the discussion of Proposition 1 that lowering $E(r)$ would cause λ_j to fall as per Eq. (3), necessitating a higher λ_k to satisfy the aggregate feasibility condition for interbank liquidity. Bank k would therefore have to forego some investment in the long-term project to set a lower $E(r)$. All else constant, a liquidity floor limits j's ability to decrease λ_j. This changes the responsiveness of λ_j to $E(r)$, which in turn changes the tradeoffs to k of setting a lower $E(r)$. In the extreme case where λ_j cannot fall below $\alpha = \lambda_j^*$, bank k would clearly want to set a lower $E(r)$, i.e., k would deviate from the unregulated equilibrium even though this equilibrium remains feasible after the introduction of the liquidity floor. The profitability of this deviation reflects the fact that an (uncircumventable) floor $\alpha = \lambda_j^*$ would enable bank k to pay a lower interest rate in the state where it needs to borrow without changing the amount it can borrow. Naturally, the relevance of this extreme case depends on the properties of the shadow technology. The condition $\frac{h''(0)}{(h'(0))^3} \to 0$ is effectively a statement that $h(\xi_i)$ is steep for low $\xi_i > 0$, in which case the liquidity floor is not a fortress against declines in λ_j. Lower $E(r)$ would then be met by sufficiently lower λ_j to eliminate any incentive of k to deviate from the unregulated equilibrium when this equilibrium is feasible.

threshold $\overline{\alpha} > \lambda_j^*$, where $\overline{\alpha} \to \lambda_j^*$ as $h'(0) \to \infty$ and $\frac{h''(0)}{(h'(0))^3} \to 0$. In other words, when $h'(0) < \infty$, bank k increases $E(r)$ to keep $\xi_j = 0$ as α is pushed above λ_j^*, but this stops being optimal at the threshold $\overline{\alpha}$.

Next, we explore the incentives of k to operate the shadow technology:

Lemma 4 *(No shadow banking by interbank price-setter). There exists a* $\overline{\delta}_1 > 0$ *such that, in the vicinity of* $\alpha = \lambda_j^*$, *bank* k *will optimally choose* $\xi_k = 0$ *for any competition parameter* $\delta_1 \in \left(0, \overline{\delta}_1\right)$ *in Eq. (7).*

The intuition for Lemma 4 follows from $\lambda_k^* > \lambda_j^*$ in the unregulated equilibrium. Bank k is not constrained by regulation as α is perturbed above λ_j^* so the only incentive to set $\xi_k > 0$ is to offset the decrease in funding share x_k caused by $\xi_j > 0$. But, unless δ_1 is very large, the benefit to k of offsetting the reduction in x_k that occurs in the vicinity of $\alpha = \lambda_j^*$ is smaller than the marginal cost of using the shadow technology to compete with j, hence k will set $\xi_k = 0$. In practice, there may be other ways for k to compete, but as long as they are also costly, adding them to the model will not change the result in Lemma 4 that k allows x_k to fall as α is pushed above λ_j^*.[8]

3.4 Credit Boom Result

Taken together, Lemmas 3 and 4 imply the following credit boom result:

Proposition 3 *(Equilibrium credit boom). For* $\delta_1 \in \left(0, \overline{\delta}_1\right)$, *as the liquidity floor* α *is perturbed above* λ_j^*, *total credit rises if and only if* $s' = B$.

[8] It is useful to emphasize here the two different reasons why an action $\xi_i > 0$ may be taken: regulatory arbitrage and competition for funding. The first is pure shadow banking; the second is not. Suppose we decouple the technology $h(\cdot)$ so that bank i can take an action ξ_i to have the *option* of moving fraction $h(\xi_i)$ of its funding x_i off balance sheet, with the fraction of $h(\xi_i)\, x_i$ that is actually moved modeled as a separate choice $\tau_i \in [0, 1]$. The effective regulatory constraint is then $\lambda_i \geq \alpha\,(1 - \tau_i h(\xi_i))$. Bank k would be indifferent between any $\tau_k \in [0, 1]$ while j would unambiguously choose $\tau_j = 1$ as α is perturbed above λ_j^*.

Total credit in the model economy is the total amount of funding (normalized to 1) less the liquidity held by the banking sector, i.e.,

$$LIQ \equiv \lambda_j (1 - x_k) + \lambda_k x_k$$

A credit boom is thus equivalent to a reduction in LIQ. From Lemma 1, we know that aggregate feasibility will hold with equality in the state with the highest aggregate withdrawals, s'. We can thus write

$$LIQ = \Theta^{s'} (x_k)$$

in any equilibrium, where

$$\Theta^{s'} (x_k) \equiv \theta_j^{s'} + \left(\theta_k^{s'} - \theta_j^{s'} \right) x_k$$

Note that LIQ represents the aggregate supply of liquidity in the interbank market at $t = 1$ while $\Theta^{s'} (x_k)$ represents the aggregate demand for liquidity in state s'. From Lemmas 3 and 4, a liquidity floor α that binds infinitesimally on the unregulated equilibrium leads ξ_j to rise relative to ξ_k, lowering x_k as per Eq. (7). This implies a fall in $\Theta^{s'} (x_k)$, and thus a reduction in the equilibrium supply of liquidity LIQ, if and only if $\theta_k^{s'} > \theta_j^{s'}$. The condition $\theta_k^{s'} > \theta_j^{s'}$ is equivalent to the condition $s' = B$ since B is defined as the state where k gets a higher liquidity shock than j.

The credit boom can also be decomposed directly from the supply side. In particular,

$$\frac{dLIQ}{d\alpha} = (1 - x_k) \frac{d\lambda_j}{d\alpha} + x_k \frac{d\lambda_k}{d\alpha} + (\lambda_k - \lambda_j) \frac{dx_k}{d\alpha}$$

The main conceptual channel for the credit boom is the third term, which reflects (i) $\lambda_k > \lambda_j$ in the unregulated equilibrium as per Proposition 1 and (ii) $\frac{dx_k}{d\alpha} < 0$ as a result of the perturbation in Lemmas 3 and 4. In words, the liquidity floor triggers a reallocation of funding towards the less liquid (and hence constrained) banks as they engage in shadow banking to loosen the constraint. We refer to this as the reallocation channel.

The sign of $\frac{d\lambda_j}{d\alpha}$ will depend on parameters. The direct effect of the regulation is to increase λ_j, but the shadow banking activities of j will be a countervailing force. The sign of $\frac{d\lambda_k}{d\alpha}$ will also depend on parameters. Even though k is not constrained by the regulatory perturbation, the equilibrium is changing so k may strategically choose to become more or less liquid. We explore this further in Proposition 4 and in the quantitative analysis.

Notice from Proposition 3 that j and/or k will increase liquidity by enough to eliminate the credit boom implied by the reallocation channel if and only if $s' = A$, i.e., if and only if aggregate feasibility binds in the state where k (the interbank price-setter and hence the more liquid bank) is a net lender. This requires k to have a small enough initial funding share, namely $x_k^0 < \underline{x}_k^0$. For larger x_k^0, the feasibility constraint is tighter in state B, i.e., $s' = B$, and the credit boom occurs.

3.5 Additional Results

Next, we establish some additional properties of the regulated equilibrium when interbank markets are characterized by pricing power. We focus specifically on a set of predictions that can emerge from our framework but not from a framework without interbank market power. These predictions will help identify the role of interbank market power in our quantitative application (Section 5).

Proposition 4 *(Additional results and necessity of interbank market power). When k has interbank market power, the following can co-exist as α is perturbed above λ_j^*: (i) credit boom, (ii) convergence of on-balance-sheet liquidity ratios, (iii) corr $\left(\xi_j, E\left(r\right)\right) > 0$. These three features cannot hold simultaneously if all banks are price-takers, even if $g\left(\cdot\right)$ is allowed to differ across banks.*

We have already established that the regulation considered in the statement of Proposition 4 generates a credit boom in our model if and only if $s' = B$ (see Proposition 3). This boom involves an increase in ξ_j, i.e., the emergence of shadow banking among the price-taking banks, so to understand *corr* $\left(\xi_j, E\left(r\right)\right) > 0$, we must understand why the price-setter k elects to increase $E\left(r\right)$ as the regulation is introduced.

The expected interbank rate $E(r)$ has several effects on the profits of k. First is the direct effect on k's interbank borrowing costs. We recall $r_A = \overline{r}_A$, so increases in $E(r)$ reflect increases in r_B, where B is the state in which k borrows from the interbank market to meet withdrawal shocks at $t = 1$. The second effect of $E(r)$ on k's profit works through the regulatory arbitrage motive of the price-taking banks. There is less incentive to circumvent a liquidity floor when liquidity is expected to be expensive. The optimal shadow banking action of the price-taking banks, ξ_j as characterized in Lemma 2, is thus decreasing in $E(r)$. This has two implications. First, the liquidity ratio λ_j of these banks is increasing in $E(r)$ over the range of interbank rates where the liquidity floor is a binding constraint; see Lemma 2, specifically the range $E(r) \leq \underline{R}(\alpha)$. Intuitively, a decrease in ξ_j implies less circumvention of the liquidity floor and thus a higher liquidity ratio among constrained banks. Second, a decrease in ξ_j implies an increase in k's funding share x_k as per Eq. (7) and thus a decrease in the funding share $x_j \equiv 1 - x_k$ of the price-takers.

Taken together, the increases in λ_j and x_k that stem from the weakened regulatory arbitrage motive of the price-takers as $E(r)$ increases have an ambiguous effect on the size of k's interbank trades, $\left(\theta_j^s - \lambda_j\right)(1 - x_k)$ in state $s \in \{A, B\}$. However, the increase in k's funding share x_k increases the total return that k can earn from the long-term investment project, $g(1 - \lambda_k)\, x_k$. The price-setting bank thus increases $E(r)$ to stop the price-taking banks from encroaching heavily on its funding share through their regulatory arbitrage activities. This is a form of asymmetric competition, wherein the price-setter uses its price impact on the interbank market to fend off competition from the price-takers and their off-balance-sheet activities, instead of directly competing with them by increasing ξ_k relative to ξ_j. The result is that both ξ_j and $E(r)$ rise in response to the tightening of liquidity regulation.

Consider next the on-balance-sheet liquidity ratios of the two types of banks. The true liquidity ratio of the price-taking banks is $\lambda_j = \alpha\left(1 - h\left(\xi_j\right)\right)$ for $\alpha \geq \lambda_j^*$, where $h\left(\xi_j\right)$ is the fraction of funding moved off-balance-sheet via shadow banking, but the on-balance-sheet liquidity ratio of these banks is α, which increases as a result of perturbing α above λ_j^*.

In contrast, the price-setting bank's on-balance-sheet liquidity ratio is the same as its true liquidity ratio, λ_k, since the price-setter is not constrained by a liquidity floor in the vicinity of λ_j^*. The proof of Proposition 4 establishes that the increase in $E(r)$ by k to defend its funding share elicits enough additional liquidity from the price-takers relative to the demand for liquidity in state B that k can decrease λ_k in favor of the long-term project. Accordingly, there is convergence in the on-balance-sheet ratios of the two types of banks.

The discussion here can be tied back to the decomposition of the credit boom result in Section 3.4. Although k increases $E(r)$ to temper the decrease in x_k that results from ξ_j, it also decreases λ_k. The credit boom in Proposition 4 then reflects a reallocation of funding towards the less liquid banks that survives asymmetric competition, $\frac{dx_k}{d\alpha} < 0$, as well as a decrease in the liquidity ratio of the more liquid bank, $\frac{d\lambda_k}{d\alpha} < 0$.

The last part of Proposition 4 highlights the role of interbank market power. To get a credit boom without interbank market power, it must be the case that banks are ex ante heterogeneous on some other dimension, e.g., productivity in the long-term project. If all banks were ex ante identical, then the aggregate feasibility condition would eliminate the possibility of a credit boom. In particular, α would elicit the same response ξ_i from all banks, so the funding shares x_i and thus the total demand for liquidity at $t = 1$ would not change, meaning $E(r)$ would adjust to keep aggregate liquidity constant. This is a competitive market at work. Ex ante heterogeneity is therefore necessary for a credit boom, and interbank market power is clearly a form of ex ante heterogeneity. To appreciate its role in our results, consider instead a model with all price-taking banks that differ on $g(\cdot)$. The more productive banks would choose a lower liquidity ratio than the less productive banks in an unregulated equilibrium and would respond to a binding liquidity floor by engaging in shadow banking. This can increase the funding share of productive banks relative to unproductive ones and lead to a credit boom by the reallocation channel discussed in Section 3.4. However, since the unconstrained banks are price-takers as well, their liquidity ratio will only fall if liquidity is expected to be less expensive. It is thus impossible to have all three results listed in Proposition 4 in the absence of interbank market power.

4 Discussion and Extensions

This section discusses inefficiency in the unregulated equilibrium and conditions under which a liquidity regulation α would in fact be optimal (Section 4.1). We also discuss the robustness of our credit boom result to an extension with equity issuance by banks (Section 4.2) and explain how a central bank could achieve constrained efficiency when the conditions for a credit boom exist and render the regulation suboptimal (Section 4.3). Finally, we discuss parallels between our model and the National Banking Era in the U.S. (Section 4.4).

4.1 Motivation for Regulation

The analysis so far has been agnostic about the case for regulatory intervention. We now explore the liquidity choices of a social planner as a function of x_k^0. There are many considerations behind the socially optimal size distribution of banks; our goal is not to make statements about them so we solve the planning problem conditional on x_k^0.

To broaden the scope of the discussion, we allow for the possibility of a third state that occurs with probability $\varepsilon \in [0, 1)$, where $\pi_A + \pi_B + \varepsilon = 1$. In this third state, all savers want to withdraw all funding from all banks at $t = 1$. This is a complete run on the banking system and banks cannot honor all withdrawal requests at $t = 1$. Because of limited liability, all banks make zero profit in this third (crisis) state so their optimization problems are unchanged from before.[9] However, there is a social cost of not honoring all withdrawals at $t = 1$ in the crisis state. The social cost is captured by a function $\kappa(\cdot)$ which has properties $\kappa(0) = 0$, $\kappa'(\cdot) > 0$, and $\kappa''(\cdot) > 0$. Its argument is the fraction of withdrawals that cannot

[9] Limited liability can take the form of each bank honoring withdrawals up to its reserves at $t = 1$ then transferring its long-term project to a receiver. Projects are illiquid, so the receiver simply returns all the proceeds at $t = 2$ to savers who could not withdraw at $t = 1$. If savers neglect tail risks and/or engage in local thinking along the lines of Gennaioli and Shleifer (2010), then Eq. (7) and $h'(\cdot) > 0$ as derived in Online Appendix D are unaffected by the introduction of small $\varepsilon > 0$.

be honored at $t = 1$ with the total liquidity available in the banking system.

The social planner chooses λ_j and λ_k to maximize

$$g(1-\lambda_j)\left(1-x_k^0\right)+g(1-\lambda_k)\,x_k^0-\varepsilon\kappa\left(1-\lambda_k x_k^0-\lambda_j\left(1-x_k^0\right)\right)$$

subject to aggregate feasibility in both non-crisis states, A and B.

Proposition 5 *(Inefficiency in decentralized equilibrium). Consider $s' = B$. Aggregate liquidity in the decentralized equilibrium is inefficiently low if $\varepsilon > 0$ and*

$$\kappa'\left(1-\Theta^B\left(x_k^0\right)\right) > \frac{g'\left(1-\Theta^B\left(x_k^0\right)\right)}{\varepsilon} \tag{12}$$

It is efficient otherwise, but the distribution of liquidity across banks is always inefficient.

In Proposition 3, we considered values of x_k^0 such that $s' = B$. We consider the same values here. Intuitively, the planner wants perfect risk-sharing, $\lambda_j = \lambda_k$, while the decentralized equilibrium achieves $\lambda_j < \lambda_k$ because banks differ in interbank market power (Section 2.4). The distribution of liquidity across banks is therefore inefficient. Whether aggregate liquidity is also inefficient depends on whether it is socially optimal to have excess liquidity in both non-crisis states. If the social cost of not honoring withdrawals in a crisis is sufficiently steep, i.e., if (12) holds, then the planner will indeed forego some investment in the long-term project to have more liquidity available in the crisis state. Aggregate feasibility will then be slack in both non-crisis states when evaluated at the planner's solution. In the decentralized equilibrium, however, aggregate feasibility binds in the non-crisis state with the most withdrawal pressure (Lemma 1). Banks do not internalize the social cost of not honoring withdrawals in a crisis, and, with limited liability, make zero profit if this state occurs. Aggregate liquidity is therefore inefficiently low.

The inefficiency just discussed opens the door for regulation. In addition to (12), which is necessary and sufficient for the planner to want to boost aggregate liquidity in the decentralized equilibrium, we assume $\varepsilon\kappa'\left(1-\min\left\{\theta_k^B,\theta_j^A\right\}\right) < g'\left(1-\min\left\{\theta_k^B,\theta_j^A\right\}\right)$ so that there is an active interbank market in both non-crisis states at the planner's solution, i.e.,

$\lambda^* < \min\left\{\theta_k^B, \theta_j^A\right\}$, where λ^* denotes the planner's solution and solves $\frac{g'(1-\lambda^*)}{\kappa'(1-\lambda^*)} = \varepsilon$.[10] The rest of this section explores the implementability of the planner's solution, $\lambda_j = \lambda_k = \lambda^*$, via a liquidity floor.

Proposition 6 *(Optimal liquidity floor without shadow banking). Consider* $\widetilde{\pi}\overline{r}_A \leq \overline{R}(\lambda^*)$*. If there is no shadow banking technology, i.e.,* $h(\cdot) = 0$*, then a liquidity floor* $\alpha = \lambda^*$ *implements the planner's solution.*

Proposition 6 establishes the existence of a liquidity floor α that implements the planner's solution in the absence of shadow banking. This floor (i) binds on the representative interbank price-taker j and (ii) prevents the price-setter k from decreasing λ_k below λ^* to make aggregate feasibility hold with equality as λ_j increases to λ^*. Both banks are then constrained to hold $\lambda_j = \lambda_k = \lambda^*$.[11]

Proposition 7 *(Effect of interbank market power on optimal liquidity floor with shadow banking). When the shadow banking technology exists, there is a liquidity floor* $\alpha > \lambda^*$ *that implements the planner's solution in a competitive equilibrium. With interbank market power, however, such an* α *may not exist and a liquidity floor* $\alpha > \lambda_j^*$ *may be welfare-reducing.*

The first part of Proposition 7 establishes the existence of a liquidity floor α that implements the planner's solution if shadow banking exists but the price-setter's interbank market power is removed. The optimal α exceeds the desired outcome λ^* because the planner accounts for the use of shadow banking to circumvent regulation. He thus sets $\alpha > \lambda^*$ to achieve $\lambda_i = \lambda^*$ after the rise of shadow banking is taken into account.

A similar policy is much more dubious in the presence of interbank market power, as indicated by the second part of Proposition 7. With both interbank market power and

[10] See the proof of Proposition 5 for the formal derivation.

[11] The restriction on $\overline{r}_A$ in the statement of Proposition 6 explicitly prevents k from setting r_A so high as to incentivize $\lambda_j > \lambda^*$. If we remove this restriction, then some mild conditions on the curvature of $g(\cdot)$ would be needed to conclude that k does not find it optimal to choose an r_A that delivers $\lambda_j > \lambda^*$. This is discussed further in the proof, but, broadly speaking, the planner's solution can be implemented via liquidity regulation in the absence of shadow banking.

a shadow banking technology, perturbing α above the unregulated equilibrium λ_j^* triggers a credit boom (Proposition 3). This boom implies a less liquid banking system, which exacerbates the social cost in the crisis state and reduces welfare. Consider now values of α beyond the perturbation. Condition (12) implies that the planner's solution cannot be implemented if Lemma 1 holds with $x_k \leq x_k^0$ so, if there exists a liquidity floor that achieves the efficient level of aggregate liquidity, the price-setter k has to be constrained.[12] All banks would then be constrained because it is not optimal for k to set $E(r)$ so high that j is unconstrained. Thus, $\lambda_i = \alpha(1 - h(\xi_i))$ for $i \in \{j, k\}$. The planner's solution requires perfect risk-sharing, $\lambda_j = \lambda_k$, which means all banks must be equally constrained so that they take the same shadow banking action ξ_i. But then k, who sets $E(r) > 0$, cannot have the same liquidity ratio as j unless the shadow price of liquidity is positive. Lemma 1 therefore holds with $x_k = x_k^0$, contradicting the premise that the efficient level of aggregate liquidity is achieved.

We have restricted attention here to a liquidity floor α that is common to all banks; the regulator could also consider different liquidity floors for different banks, α_i. For example, imposing a sufficiently higher liquidity floor on the interbank price-setter as compared to the price-takers could increase aggregate liquidity even as banks attempt to circumvent the regulation. Intuitively, the funding share x_k will rise if the price-setting bank is constrained enough that it engages in more shadow banking than the price-takers, $\xi_k > \xi_j$. With $s' = B$, the maximum demand for liquidity outside of the crisis state will then also rise and have to be met in equilibrium by a higher aggregate supply. We sketch out this possibility at the end of the proof of Proposition 7.

In practice though, we do observe common liquidity floors in many countries. The broader lesson that emerges from this section is that regulators should think twice about using a simple liquidity floor α when the interbank market is not competitive and banks have access to a shadow banking technology. It is the combination of interbank market power and shadow

[12] Recall that we consider values of δ_1 such that bank k does not take the shadow action simply to increase its funding share relative to the unregulated equilibrium.

banking that is problematic, as this section has shown α to be optimal otherwise. In Section 5, we explore the quantitative importance of the credit boom that results from tightening a simple liquidity regulation in an environment with these two features.

4.2 Bank Capital

Our main analysis focuses on banks funded entirely by callable liabilities, i.e., funding that can be withdrawn at $t = 1$, to show most transparently that a liquidity floor can lead to an unintended credit boom. In reality, banks are also partly funded by equity, which is costly to issue but not callable. The model can be extended to allow for equity issuance by banks.

In particular, consider that bank i has debt funding x_i as well as equity funding e_i, which it raises at a cost $\tau(e_i)$. The liquidity ratio λ_i is calculated as a fraction of the bank's total funding, now $x_i + e_i$, so the aggregate feasibility condition becomes

$$\lambda_k (x_k + e_k) + \lambda_j (x_j + e_j) \geq \theta_k^s x_k + \theta_j^s x_j$$

for each $s \in \{A, B\}$, and the effective liquidity requirement is

$$\lambda_i (x_i + e_i) \geq \alpha (1 - h(\xi_i)) x_i$$

instead of Eq. (6). We can also allow for a risk-weighted capital requirement

$$e_i \geq \beta [(1 - \lambda_i)(x_i + e_i) - h(\xi_i) x_i]$$

which involves a risk weight of zero on reserves and requires bank i's equity to be at least a fraction β of its on-balance-sheet illiquid assets. Notice that the risk-weighted capital requirement implicitly imposes a liquidity floor,

$$\lambda_i \geq \frac{1 - h(\xi_i) - \left(\frac{1}{\beta} - 1\right)\frac{e_i}{x_i}}{1 + \frac{e_i}{x_i}}$$

and can thus be counter-productive in the same way we found liquidity requirements to be counter-productive in the main analysis.

Online Appendix F formalizes the extended model sketched here. The forces behind our credit boom result also arise in the model with equity, both qualitatively and in a recalibrated version of the policy experiment considered in Section 5. The policy experiment in the extended model also reveals that tightening α endogenously loosens capital requirements by incentivizing banks to move less liquid assets off the balance sheet.

4.3 Central Bank Liquidity Injections

One way to implement the planner's solution without setting off the type of shadow banking explored in our paper is to have a central bank intervene in the interbank market by (i) being extremely responsive to any deviation from a pre-defined interest rate target and (ii) setting the target high enough to price in the social cost of insufficient liquidity in the crisis state. Formally, suppose the central bank announces a target interbank rate r^* and state-dependent liquidity injections $\psi\left(r_s - r^*\right)$, where $\psi > 0$. Injections can be positive or negative, and we assume the central bank is credible. Aggregate feasibility for each $s \in \{A, B\}$ is then

$$\left(\lambda_j - \theta_j^s\right)\left(1 - x_k^0\right) + \left(\lambda_k - \theta_k^s\right) x_k^0 + \psi\left(r_s - r^*\right) \geq 0 \tag{13}$$

The problem of bank j is unchanged from the unregulated environment, i.e., λ_j still depends on $E\left(r\right)$ as per Eq. (3). The problem of bank k is also unchanged except that Eq. (13) replaces Eq. (1) as a constraint. The following proposition establishes the implementability of the planner's solution via interest rate targeting:

Proposition 8 *(Implementing constrained efficiency without liquidity regulation). As* $\psi \to \infty$*, setting* $r^* = g'\left(1 - \lambda^*\right) - 1$ *achieves the planner's solution.*

The higher is ψ, the less control k has over the interbank rate. Intuitively, a high rate will prompt a larger liquidity injection by the central bank, bringing down the rate that prevails in equilibrium. In the limit, bank k becomes a price-taker, i.e., it chooses $r_s \to r^*$ as $\psi \to \infty$, and both λ_k and λ_j are pinned down by Eq. (3) with $E\left(r\right) = r^*$, delivering the planner's solution for the appropriate r^*.

Online Appendix G studies the limiting case of $\psi = \infty$ in more detail. We find that liquidity regulation can lead to a credit boom in a model with interest rate targeting if (i) banks differ in their marginal returns and (ii) the central bank injects liquidity to maintain its target in the non-crisis state where the bank with higher marginal returns borrows. Thus, using a liquidity floor α instead of setting the interest rate target that achieves the efficient level of liquidity can still generate a credit boom in settings with $\psi \to \infty$, but the conditions are arguably more restrictive than the conditions for a credit boom in settings with just interbank market power. The zero lower bound also raises some interesting considerations here. If a central bank cannot (or does not) swiftly counteract spreads in relevant short-term funding markets because its traditional toolkit is constrained, market power may emerge in settings where central bank intervention had traditionally eliminated it, leading us back to our core model.

4.4 Parallels to the U.S. National Banking Era

The National Banking Era in the U.S., prior to the creation of the Federal Reserve in 1913, provides some interesting parallels to the discussion here. The comprehensive work of Sprague (1910) documents the liquidity choices of banks during this period and their evolution.

Liquid assets during the National Banking Era took the form of vault cash and demand deposits with other banks. National banks in the interior of the U.S. (country banks) deposited with national banks in major municipalities (reserve city banks), most of which also deposited with national banks in New York City (the NYC banks). The NYC banks were more liquid in terms of cash holdings. In October 1897, for example, the ratio of cash reserves to net deposits was 11.6% among country banks, 17.8% among reserve city banks, and 27% among the NYC banks (Sprague (1910), pgs. 220, 221).[13] The legal requirement for national

[13] Net deposits equals the sum of individual deposits and balances due to other banks minus the sum of

banks in NYC was 25% and the vast majority of these banks were also members of the New York Clearinghouse Association (NYCH), which influenced its members, notwithstanding legal requirements, through internal governance (Moen and Tallman (2000)). Demand deposits were typically lent on call to stock brokers in NYC. According to Moen and Tallman (2014), the NYC banks considered the "external effects" of liquidating call loans to meet withdrawal requests, in contrast to others who made individually small loans to this market. A similar view is advanced by Sprague (pgs. 45, 62, 269, 301), providing insight into the higher cash ratios of the NYC banks. Our model with interbank market power provides a way of formalizing the narratives in the literature. If the NYC banks influenced interbank rates, broadly defined, then they would internalize the effect of their liquidity and indeed be more liquid.

By August 1907, the cash ratios of the country and reserve city banks had fallen to 7.6% and 13.4% respectively in favor of demand deposits (pg. 220). The net deposits of all national banks more than doubled from 1897 to 1907, leading to a near doubling of demand deposits held in NYC (pgs. 218, 222). Interestingly, growth in the overall deposits of the NYC banks was only 63% over this period (pg. 221). The cash ratio of the NYC banks stayed roughly stable, but, in light of the increase in demand deposits from the interior and the fact that such deposits were known to be flighty, a stable cash ratio meant that the NYC banks were effectively less liquid in 1907 than they had been in 1897 (pgs. 222, 226, 236).

The relative weakness in overall deposit growth among the NYC banks, as well as their lower effective liquidity, coincided with a rapid expansion of trust companies in New York. Loans by trust companies more than tripled from 1897 to 1907, rivaling the lending volume of the national banks in NYC (pg. 227). Trust companies were largely unregulated and held notoriously little cash (pg. 226). Accordingly, they could afford to offer very high interest rates to attract deposits (pg. 255). In 1903, the NYCH announced that trust companies had to become more liquid in order to continue clearing through a NYCH member. The trusts

clearinghouse exchanges, bills of other banks, and balances due from other banks.

responded by surrendering clearinghouse privileges so as to continue aggressively taking deposits and making loans (pg. 253). This poached funding away from the NYC banks, yet the latter still increased lending by 77% between 1897 and 1907 (pg. 221), exceeding the growth in their deposits. The rapid expansion of shadow banking and the decline in the deposit share of the NYC banks in response to the tightening of liquidity standards is exactly what our model would predict. The decrease in the effective liquidity of the NYC banks is also predicted by our model as an endogenous response of the price-setter.

In October 1907, a run developed against Knickerbocker Trust when depositors became worried about its connection to the companies of C.F. Morse, a director at several banks of moderate size in NYC and a businessman whose practices were generally distrusted (pgs. 248, 251). Banks around the country then began withdrawing demand deposits from NYC in a panic, and the NYC banks responded by suspending convertibility to maintain cash ratios. Sprague contends that the NYC banks neglected the economic costs to the interior of suspension (pgs. 280, 319). Alternatively, if maintenance of cash ratios was so important, the entire system should have been more liquid. The aggregate inefficiency was therefore in the direction of insufficient liquidity (see also Farhi, Golosov, and Tsyvinski (2007, 2009) on this interpretation). Our model suggests that uniformly higher reserve requirements would not have remedied the situation. Instead, such requirements could have triggered further dissociation from the NYCH, e.g., by less liquid state banks, in favor of trust-like activities, which would have shifted funding shares and reduced aggregate liquidity even more.

5 Quantitative Analysis

We have focused so far on qualitative predictions of the theory. We now want to study quantitative implications. We choose China as the setting for our quantitative analysis. In addition to being one of the world's largest economies, China experienced a near doubling of its debt-to-GDP ratio over the past decade, along with unprecedented growth in its ratio of private credit to private savings. Our model predicts that some credit booms are unintentionally caused by liquidity regulation so we are interested to know whether liquidity regulation can account for at least part of the Chinese experience.

Liquidity rules in China involve reserve requirements and, until late 2015, a loan-to-deposit cap. The loan-to-deposit cap was introduced in 1995 to prevent banks from lending more than 75% of the value of their deposits to non-financial borrowers. The remaining 25% had to be kept liquid, with reserve requirements dictating how this liquidity was to be divided between pure reserves and other liquid assets. In practice, enforcement of the 75% loan-to-deposit cap was lax until 2008, when the China Banking Regulatory Commission (CBRC) announced a tougher stance and began increasing the frequency of its loan-to-deposit monitoring. The enforcement action began with CBRC monitoring the end-of-year loan-to-deposit ratios of all banks more carefully. CBRC then switched to monitoring end-of-quarter ratios in late 2009, end-of-month ratios in late 2010, and average daily ratios in mid-2011. The increasing frequency of CBRC's loan-to-deposit enforcement was also complemented by a rapid increase in the reserve requirements set by the central bank.[14]

Interbank market power was central to our theory of unintended credit booms. Section 5.1 establishes that large commercial banks in China impact the interbank market to a much greater extent than small commercial banks. Section 5.2 then calibrates the model to Chinese data. We use the calibrated model to study how large a credit boom our model can produce (Section 5.3) and present a structural estimation to evaluate the importance of various shocks (Section 5.4).

[14] We refer the reader to Hachem (2018) and Song and Xiong (2018) for more on China's regulatory environment and financial institutions. It is useful to note that capital regulation in China follows international standards. Chinese banks were comfortably above minimum capital requirements as CBRC tightened enforcement of the loan-to-deposit cap. China also increased capital requirements in 2013 and 2014, in line with Basel III, but bank capital ratios remained unconstrained. We discuss this further in Online Appendix I which accompanies Sections 5.2 and 5.3.

5.1 Empirical Evidence on China's Interbank Market

We begin with some brief institutional background on China's commercial banks and their interbank activities before presenting our empirical evidence on interbank market power.

5.1.1 Institutional Background

The Chinese economy is served by both big and small banks. The small banks include twelve joint-stock commercial banks (JSCBs) which operate nationally, as well as over two hundred city banks operating in specific regions. Many rural banks have also emerged.[15] The JSCBs are typically larger than the city and rural banks but all of these banks are still individually small when compared to China's big banks (the Big Four). The Big Four are the four commercial banks established by the central government after the Cultural Revolution. Market-oriented reforms initiated in the 1990s made the Big Four almost entirely profit-driven and removed government involvement from day-to-day operations. However, a legacy of minimal competition between these four banks remains. China's banking sector is therefore well approximated by a model with one big bank and many small banks.

We characterize the market structure and the relative importance of the Big Four in China's interbank repo market in Online Appendix H.[16] In addition to the Big Four, the JSCBs, and other smaller players, China has three policy banks which participate in the interbank repo market. The policy banks are not commercial banks. Instead, they raise money on bond markets and take directives from the central government about where to invest. The policy banks and the Big Four are the central lending nodes in China's interbank repo market. Detailed analysis of a dramatic spike in interbank interest rates on June 20, 2013 demonstrates that policy banks provided generous amounts of liquidity but interbank rates did not fall because the Big Four were extremely restrictive. This is a concrete example

[15] There were 951 commercial banks in China's business registration records at the end of 2014. Of these, 438 were established after 2007 and reflected mainly the conversion of rural credit cooperatives into rural banks rather than the entry of new credit providers.

[16] China has both an interbank repo market and an uncollateralized money market. We focus on the repo market since it is vastly larger.

of the ability of the Big Four to drive pricing in the interbank market.

The spike in June 2013 eventually led Chinese regulators to announce that the interbank liabilities of a commercial bank should not exceed one-third of its total liabilities. The goal was to increase transparency and facilitate supervision, not to suppress interbank activity.[17] Notably, negotiable certificates of deposit (NCDs) were exempt from the definition of interbank liabilities, leading to rapid growth in NCD issuance (Gu and Yun (2019)). We use NCDs in the empirical strategy below, so it will be useful to overview the data here.

Daily average NCD issuance increased from less than RMB 1 trillion in 2014 to roughly RMB 13 trillion in 2016, before stabilizing around RMB 20 trillion in 2017 and 2018. This is still modest relative to the interbank repo market, whose daily average volume is about 28 times that of NCD issuance. Nevertheless, interest rates on NCDs track fluctuations in the overnight repo rate; the correlation ranges from 0.67 for 1-month NCDs to 0.78 for 6-month NCDs.[18] Roughly 75% of NCDs have a maturity of 3 months or longer, whereas overnight transactions account for more than 80% of total trading volume on the interbank repo market. On an annualized basis, NCD rates exceed repo rates, so it is not profitable to borrow via NCD issuance in order to increase repo lending.

When using NCD data, we focus on the trading days from the beginning of 2016 to the end of 2018. The NCD market was not fully developed until late 2015 and CBRC began regulating NCDs very differently after 2018 (Gu and Yun (2019)). Our sample period consists of more than 70,000 NCD announcements. The vast majority of NCDs from 2016 to 2018 were issued by small banks. The Big Four only accounted for 2.8% of the total issuance volume. Within the Big Four, Agricultural Bank of China (ABC) issued the most NCDs, followed by China Construction Bank (CCB).[19] The average size of an NCD issue was RMB

[17] See "Document No. 127: The Official Document for Interbank Businesses," an investigative report by a finance journalist (Xi You, Caijing, June 4, 2014), for how the events of June 20 triggered intensive policy discussions and eventually led to Document No. 127, "Notice on Regulating the Interbank Business of Financial Institutions," jointly issued by CBRC, the central bank, and several other agencies.

[18] While we view these correlations as reasonably high, they are lower than the correlation between the Fed Funds Rate and CD rates in the U.S., reflecting the more volatile nature of the interbank rate in China as compared to the U.S. (the latter is targeted by the central bank; the former is not).

[19] Between 2016 and 2018, ABC and CCB each accounted for 0.6% of the number of NCD issuances. Bank

2.4 billion for ABC and RMB 1.8 billion for CCB, comparable to the average size of an NCD issue by a JSCB. Thus, it is the frequency of issuance that differs between the Big Four and the JSCBs, not the size of the average issuance.

5.1.2 Empirical Evidence

We use NCDs to provide more direct evidence of the interbank market power of the Big Four. Ideally, we would like to show that the interbank repo rate responds to variations in the net liquidity supply of the Big Four, specifically variations that are not driven by the repo rate itself. Such variations are not directly observable, but the NCD market can help provide some identification, as described next.

Advance information disclosure is mandatory for each NCD issuance. The issuing bank has to announce the volume being issued, the promised interest rate, and the maturity of the product at least one trading day before issuance occurs. Our identifying assumption is that an exogenous increase in the liquidity demand of a bank leads it to both reduce lending on the interbank repo market and announce NCDs, conditional on NCD issuance being a regular channel for the bank to raise funds. The advantage of the NCD data is that announcements are made at the end of the trading day. An unanticipated NCD announcement would thus signal an impending, i.e., next trading day, decrease in the net liquidity supply of the issuing bank on the interbank repo market.

We construct a daily NCD announcement dummy for the Big Four, $NCD_{B4,t}$, which equals one if any bank in the Big Four announced NCDs at the end of trading day t. Our key specification is a regression of the daily average repo rate at date t on the NCD announcement dummy for the Big Four at date $t-1$. The repo rate is highly persistent, so we include two lags as regressors to control for dynamics in the interbank repo market prior

of China (BOC) accounted for 0.2% while Industrial and Commercial Bank of China (ICBC) did not issue more than a handful of NCDs. In terms of market volume, ABC and CCB accounted for 1.4% and 0.9% respectively while BOC accounted for 0.5%. If the variations in liquidity demand are similar among the Big Four, one would infer that ICBC and BOC have much higher costs of issuing NCDs than ABC and CCB and thus prefer to borrow from the repo market. In contrast, NCD issuance is a more viable choice for ABC and CCB to raise liquidity.

to NCD announcement.[20] We also include the required reserve ratio to control for policy changes by China's central bank (PBOC). The regression equation is thus

$$repo_t = \beta_0 + \beta_1 NCD_{B4,t-1} + \beta_2 repo_{t-1} + \beta_3 repo_{t-2} + \beta_4 RRR_t + \varepsilon_t \tag{14}$$

with β_1 as the coefficient of interest. Note that NCD announcements, if unanticipated, should have no effect on the repo rate on the announcement day. This can be confirmed by replacing $NCD_{B4,t-1}$ with $NCD_{B4,t}$ in Eq. (14) and checking that the estimated coefficient is not statistically different from zero.

We confirm that NCD announcements are in fact associated with subsequent decreases in net liquidity supply. Starting in 2015, PBOC began reporting end-of-month data on the total interbank assets and liabilities of the Big Four separately from other banks. Such data are not available at the daily frequency of NCD announcements. However, for the 36 end-of-month observations between 2016 and 2018, we can regress the Big Four's interbank asset-to-liability ratio on a dummy variable for NCD announcements on the second last trading day of the month. The estimated coefficient is -0.29 and statistically significant at the 5% level. A higher ratio of assets to liabilities on the interbank market indicates a higher net liquidity supply. The Big Four's interbank assets exceed their interbank liabilities in all 36 observations, with an average ratio of 1.91. The estimated decline of 0.29 associated with NCD announcement does not reverse the status of the Big Four as net lenders to the broader interbank market. However, it does indicate a decrease in their net liquidity supply.

Table 1 presents our main findings from the regression in Eq. (14). Column 1 shows that β_1 is positive and highly significant. The point estimate suggests that, on average, the interbank repo rate increases by 3.9 basis points after a bank in the Big Four announces NCD issuance. Replacing $NCD_{B4,t-1}$ with $NCD_{B4,t}$ in Eq. (14) delivers a statistically insignificant coefficient, consistent with the unanticipated nature of NCD announcements at the end of each trading day.

[20] All the results are robust to the number of lags; the third-and-above lags are statistically insignificant.

The effect in Column 1 is mainly driven by ABC and CCB, which accounted for more than 80% of the NCDs issued by the Big Four. This is shown in Column 2 of Table 1, which replaces $NCD_{B4,t-1}$ with a separate lagged NCD announcement dummy for each bank in the Big Four. In Online Appendix H, we run Eq. (14) using the lagged NCD announcement dummy for each JSCB; the response of the interbank repo rate to NCD announcements by the JSCBs is not statistically different from zero.[21]

Overall, then, the interbank repo rate increases in response to NCD announcements by banks in the Big Four but does not respond to NCD announcements by any of the JSCBs. Recall that the size of an average NCD issuance is similar across the Big Four and the JSCBs. Thus, the results are consistent with the Big Four's interbank market power, or, at the very least, the perception by other banks that banks in the Big Four have such market power.[22]

5.2 Calibration

We calibrate the model to China's banking sector in 2007, just before CBRC's enforcement action on the 75% loan-to-deposit cap. We introduce linear operating costs $\phi_i x_i$ and external liquidity L into the theoretical model to better fit the data. The external liquidity parameter captures the presence of non-bank financial institutions in China's interbank repo market. In the baseline calibration, L is a constant. We will allow it to vary with the interbank rate as in Eq. (13) when conducting sensitivity analysis.

We set $\alpha = 0.145$ as the initial liquidity floor, which was the required reserve ratio at the end of 2007. The average interest rate on overnight repos in China's interbank market was 2.2% per annum. The Big Four accounted for 56% of total deposits in 2007 and, as a group,

[21] As an additional robustness check, we redid the regressions in Table 1 and Online Appendix H (see specifically Table H.2) dropping all NCD announcements with a volume above RMB 10 billion. This eliminates about 5% and 2% of the NCDs issued by ABC and CCB respectively. The results are very robust.

[22] For example, if the JSCBs perceive NCD issuance by ABC as an increase in ABC's liquidity demand, they might expect higher repo rates going forward and borrow more from the repo market as soon as it opens. This collective increase in current liquidity demand would put upward pressure on the repo rate. However, the basis for the upward pressure is the expectation that ABC's liquidity demand will tighten the repo market, which is an expectation of market power.

had a loan-to-deposit ratio of 0.62, which was much lower than the JSCB ratio of 0.86.[23]

All interest rates are converted to an annualized basis for the purposes of calibration. PBOC set benchmark interest rates for traditional deposits until late 2015. The average annualized rate for short-term deposits (3 months or less) was 2% at the end of 2007. Since our model normalizes the interest rate on traditional (i.e., on-balance-sheet) funding to zero, we will deduct 2% from all interest rates used as targets in the calibration.

Benchmark loan rates were more flexible than deposit rates. The benchmark loan rate was 6.6% per annum for loans with a maturity of less than 6 months. According to PBOC's quarterly reports, about 25% and 45% of commercial bank loans had interest rates below and above the benchmark rate, respectively. The average lending rate was 7.1% and 10% of loans were charged an interest rate that was at least 50% higher than the benchmark rate. We assume a quadratic revenue function $g(1-\lambda_i) = (1+z)(1-\lambda_i) - \frac{\gamma}{2}(1-\lambda_i)^2$, where $z > \gamma$. We set $z = 0.08$ so that the highest marginal return to loans is 10%. We then set γ so that the average lending rate in the model is 7.1%.[24]

We target $E(r) = 0.2\%$ to match the average repo rate of 2.2% in 2007. We normalize $\overline{r}_A = 0$ and set $\pi_A = 0.90$, with $\pi_B = 0.09$. The target for $E(r)$ then pins down $r_B = 2.2\%$. Mapping back to the data, the interbank rate when small banks borrow is normalized to 2% and the interbank rate when the big bank borrows is 4.2%.

We calibrate x_k^0 and δ_1 to match the Big Four's market share in 2007 and 2014. Note that δ_1 is the only parameter calibrated using data from 2014. The empirical counterpart of x_k is the Big Four's share of total deposits, which is 0.56 in 2007 and 0.46 in 2014.[25]

[23] All loan-to-deposit ratios reported here are calculated using the average balances of loans and deposits during the year, not the year-end balances that are prone to manipulation. See Online Appendix I for more on the importance of using average balance data. The average loan book of each bank later became the ultimate target of CBRC's enforcement action.

[24] We are assuming the same z and γ for all banks. In practice, different banks may invest in different sectors but, adjusting for political risk, the returns are roughly comparable in China. Some anecdotal evidence can be found in Dobson and Kashyap (2006).

[25] Here, we use end-of-year deposit shares, which constitute a more general measure of bank funding than the name suggests and map well into our overall funding variable x_i. As we describe below, the relevant notion of shadow banking in China is WMPs. The majority of WMPs come due at the end of the year, at which point they automatically appear in the saver's deposit account while waiting for him/her to confirm rollover of the product.

The shadow banking technology, $h(\xi_i)$, is approximated by $\overline{h}\xi_i$, where $\overline{h} > 0$ is a constant. Our core insights do not require this technology to be strictly concave (see Online Appendix E) so we use a linear function to simplify the computational procedure. For reference, the type of shadow banking we model is well approximated by wealth management products (WMPs) in China. In 2005, the Chinese government expanded the range of financial services banks could provide. This opened the door for WMPs which represent a deposit-like product offered at endogenous interest rates. Any WMPs issued without an explicit principal guarantee do not have to be consolidated into the bank's balance sheet and are instead invested off-balance-sheet. The lack of explicit guarantees is only for accounting purposes though; there is a general perception that all WMPs are at least implicitly guaranteed by traditional banks (Elliott, Kroeber, and Qiao (2015)). Online Appendix I provides further background on the issuance of WMPs in China. WMPs were a mere 1.3% of China's total deposits in 2007. To calibrate $\overline{h}$, we target $\lambda_j = 0.14$ to match the loan-to-deposit ratio of the JSCBs in 2007 given the liquidity floor of $\alpha = 0.145$.

For the liquidity shocks, we normalize $\theta_j^B = 0$ then set $\theta_j^A = \lambda_j + 0.025$ to ensure $\theta_j^A > \lambda_j$, i.e., small banks always borrow in state A.[26] To calibrate θ_k^B, we target $\lambda_k = 0.38$ to match the loan-to-deposit ratio of the Big Four in 2007. We then set θ_k^A so that the expected liquidity shock is the same across banks, i.e., $\theta_k^A = \theta_j^A + \left(\frac{1}{\widetilde{\pi}} - 1\right)\left(\theta_j^B - \theta_k^B\right)$ where $\widetilde{\pi} \equiv \frac{\pi_A}{\pi_A + \pi_B}$.

Finally, we calibrate the operating cost parameters ϕ_j and ϕ_k to match profitability metrics in 2007, specifically a profit-to-asset ratio of 1% for the Big Four and 0.9% for the JSCBs based on financial statement data.[27]

[26] While the choice of θ_j^A is arbitrary, our results are not affected by different θ_j^A due to the slackness of the interbank market in state A. Assuming $\theta_j^A = \lambda_j + 0.25$, for instance, delivers essentially the same quantitative results.

[27] The calibrated parameters are $\gamma = 0.079$, $x_k^0 = 0.57$, $\delta_1 = 197$, $\overline{h} = 745$, $\theta_k^B = 0.62$, $\theta_k^A = 0.10$, $\phi_j = 0.031$, $\phi_k = 0.024$, $L = 0.07$. The calibrated L implies that non-bank financial institutions provide about a third of the interbank market's liquidity in state B.

5.3 Policy Experiment

We now use the calibrated model to predict what would have happened in 2014 had the only difference between 2007 and 2014 been the strength of CBRC's loan-to-deposit enforcement. To this end, we tighten the liquidity floor from $\alpha = 0.145$ to $\alpha = 0.25$, keeping all other parameters unchanged. Comparing the predicted change in the aggregate credit-to-savings ratio to the actual change observed in the data, we get an estimate of the quantitative importance of stricter liquidity rules.

The results are summarized in Table 2. Our model predicts a 42 basis point increase in the average interbank interest rate between 2007 and 2014. This is about one-third of the increase observed in the data.[28] The model also predicts a large increase in the Big Four's loan-to-deposit ratio, from 0.62 in 2007 to 0.71 in 2014. This increase is almost identical to the one in the data despite not being targeted in the calibration. Notice that stricter enforcement of the 75% cap introduced a binding constraint on China's small banks but not on the Big Four. While the Big Four were not directly affected, the sharp decline in their liquidity ratio is exactly the response predicted by our model in Proposition 4. This decline occurs as the on-balance-sheet liquidity ratio of the JSCBs rises from 0.145 to 0.25 to meet the liquidity floor. China thus experienced higher interbank rates and convergence of on-balance-sheet liquidity ratios after the tightening of liquidity regulation. We recall from Proposition 4 that interbank market power is necessary for liquidity regulation to have both of these effects alongside a credit boom.

Turning next to the credit boom, Table 2 shows that our model predicts a 6.2 percentage point increase in total credit. This is a prediction about the change in total credit relative to total savings, as total savings are normalized to 1 in the model. The relevant comparison in the data is therefore to the change in China's aggregate credit-to-savings ratio between 2007 and 2014, which we estimate to be roughly 10 percentage points.[29] The calibrated version

[28] The benchmark deposit rate was lowered from 2% in 2007 to 1.4% in 2014, so we subtract only 1.4% from the average overnight repo rate in 2014 to get the entry in the first row of column (4) in Table 2.

[29] The credit-to-savings ratio in 2007 is pinned down by the targeted values of λ_j, λ_k, and x_k. We then

of our model thus generates around 60% of China's credit boom, as measured by growth in credit over and above growth in savings, as the outcome of stricter liquidity regulation. The results are robust to introducing equity issuance by banks and a risk-weighted capital requirement (Table F.1 in Online Appendix F) as in the extended model discussed in Section 4.2.

Applying the decomposition in Section 3.4, the credit boom in Table 2 reflects

$$\underbrace{\Delta LIQ}_{-0.062} = \underbrace{\left(1 - x_k^1\right)\Delta\lambda_j}_{0.002} + \underbrace{x_k^1 \Delta\lambda_k}_{-0.040} + \underbrace{\left(\lambda_k^0 - \lambda_j^0\right)\Delta x_k}_{-0.024}$$

where λ_j^0 and λ_k^0 are the liquidity ratios at the initial regulation $\alpha^0 = 0.145$ and x_k^1 is the funding share at the tightened regulation $\alpha^1 = 0.25$. Notice that $\Delta\lambda_j$, the change in the true liquidity ratio of the interbank price-takers, is very small. It can be decomposed as

$$\Delta\lambda_j = \underbrace{\left(1 - \overline{h}\xi_j^0\right)\Delta\alpha}_{0.1014} - \underbrace{\alpha^1 \overline{h}\Delta\xi_j}_{0.0980}$$

where $\xi_j^0 \approx 0$ is the shadow banking action taken by these banks before the tightening of liquidity regulation. The first term in the expression for $\Delta\lambda_j$ captures the increase in the on-balance-sheet liquidity ratio of the price-takers to comply with tighter regulation. It is almost entirely undone by the second term, which captures the big increase in the shadow banking activities of the price-takers as they become more constrained.[30] The credit boom then reflects the reallocation of funding towards these less liquid banks ($\Delta x_k < 0$) and the strategic response of the more liquid interbank price-setter ($\Delta\lambda_k < 0$).

estimate the size of the credit boom in the data as follows. Commercial banks in China for which Bankscope has complete data collectively added RMB 40 trillion of new loans between 2007 and 2014. As a result, the ratio of traditional lending to GDP increased by 20 percentage points. The ratio of off-balance-sheet WMPs to GDP increased by 15 percentage points over the same period, which accounts for the majority of the growth in broader measures of shadow banking that can be constructed using data from China's National Bureau of Statistics (Hachem (2018)). Adding the growth of the traditional and shadow sectors, we get a 35 percentage point increase in the ratio of total credit to GDP from 2007 to 2014, which translates into a roughly 10 percentage point increase in China's credit-to-savings ratio.

[30] Online Appendix I runs panel regressions to show formally that more constrained banks engaged more heavily in shadow banking in China. We also find suggestive cross-sectional evidence that provinces with higher loan-to-deposit ratios before CBRC's enforcement action experienced more rapid financial sector growth, especially outside of traditional bank lending, after the enforcement.

The calibrated model can also be used to conduct sensitivity analysis with respect to some key parameters. The details are presented in Online Appendix J. We highlight a few of the results here. First, the emergence of a credit boom reflects interbank market power, not the large initial funding share of the Big Four. Lowering x_k^0 to the deposit share of only the largest member of the Big Four in 2007 then recalibrating the model still delivers a quantitatively important increase in credit following the tightening of liquidity rules. Second, our baseline credit boom is robust to introducing some competition between the price-taking banks for funding. In any symmetric equilibrium, $\xi_j = \overline{\xi_j}$ so the funding share of the price-takers is $x_j = 1 - x_k$, with x_k as per Eq. (7). However, off equilibrium, x_j can be increasing in the spread $\xi_j - \overline{\xi_j}$. Extending the model in this way does not change the conclusion that the effect of higher α on aggregate credit is positive. Third, increasing sufficiently the amount of external liquidity L in the interbank market dampens the size of the credit boom. The same is true if we introduce highly interest-sensitive liquidity injections by the central bank, i.e., $\psi > 0$ as in Eq. (13) with ψ large. Big inflows of liquidity from outside sources reduce fluctuations in the average interbank rate, weakening the price-setter's pricing power. A central bank that is sufficiently responsive to interbank rate fluctuations can therefore decrease the magnitude of the credit boom triggered by liquidity regulation, assuming all other parameters are held constant.

5.4 Simulation Results

We now subject the calibrated model to various shocks to see how well it matches empirical moments not targeted in the calibration. We are interested in (i) the overall ability to match these moments and (ii) the relative importance of each shock in doing so.

Table 3 reports observed correlations between the interbank repo rate and the returns to WMPs issued by small and big banks. These are the key market-determined interest rates in China and their correlations were not targeted in the calibration.

The correlations in Table 3 are calculated using monthly data from January 2008 to

December 2014. The time series for $E(r)$ is the average interbank repo rate weighted by transaction volume. The time series for ξ_j and ξ_k are the average returns promised by small and big banks respectively on 3-month WMPs, as reported in the Wind Financial Terminal.

Table 3 shows that $E(r)$ is positively correlated with ξ_j and ξ_k, as well as the spread $\xi_j - \xi_k$. It also shows that ξ_j is positively correlated with ξ_k. We would like to know the extent to which our calibrated model can replicate the correlations in Table 3. We start by considering three shocks separately: shocks to liquidity regulation, shocks to loan demand, and money supply shocks. We then simulate the model allowing for all three shocks simultaneously.

5.4.1 Shocks to Liquidity Regulation

We allow α, the parameter governing liquidity regulation, to be drawn from a normal distribution:

$$\alpha = \overline{\alpha} + \varepsilon_\alpha \tag{15}$$

where ε_α is normally distributed with mean 0 and variance σ_α^2. We set $\overline{\alpha} = 0.25$, which generates the 75% cap on the loan-to-deposit ratio. We draw values of α using Eq. (15) and simulate the model for each value to generate the average interbank rate $E(r)$ and the returns ξ_j and ξ_k offered by small and big banks respectively. We then use Simulated Method of Moments to estimate the unknown parameter σ_α. Online Appendix K describes the estimation procedure in more detail.

The first column of Table 4 reports the estimated parameter values (Panel A) and predicted correlations (Panel B). The observed correlations from Table 3 appear in the last column of Panel B. The estimated σ_α is sizable and highly significant. Also notice that the estimated model predicts the positive correlations between $E(r)$ and each of ξ_j, ξ_k, and $\xi_j - \xi_k$ as well as the positive correlation between ξ_j and ξ_k. We have explained the mechanism through which a tightening of liquidity regulation constrains banks that are price-takers on the interbank market (i.e., small banks) and incentivizes them to engage in shadow banking, which in turn incentivizes the price-setter (i.e., big bank) to defend its funding share by increasing $E(r)$. This generates a positive correlation between $E(r)$ and ξ_j; see Proposition 4. For a large enough shock to liquidity regulation, which we consider in the quantitative analysis, the price-setter also defends its funding share by increasing ξ_k away from zero. This leads ξ_k to be positively correlated with ξ_j and $E(r)$. The response of ξ_k to α is less dramatic than the response of ξ_j to α because the price-setter is not fundamentally constrained by the regulation, leading to a positive correlation between $E(r)$ and the spread $\xi_j - \xi_k$.

Shocks to α therefore generate all the right signs for the correlations between the interbank rate and WMP returns. At the same time, the predicted correlations are higher than those in the data. It will thus be useful to also allow for other shocks, as is done next.

5.4.2 Loan Demand Shocks

Shocks to loan demand are introduced by allowing the parameter z to fluctuate (recall $g'(0) = 1 + z$ in the calibrated model). Specifically:

$$z = \overline{z} + \varepsilon_z$$

where ε_z is normally distributed with mean 0 and variance σ_z^2. Loan demand shocks have their own importance in China given that fiscal stimulus was undertaken in 2009 and 2010. The stimulus package sought to combat negative spillover from the global financial crisis by providing a direct boost to aggregate demand. To the extent that stimulus increased loan demand, it did so at all banks in a largely uniform way (Bai, Hsieh, and Song (2016)). An increase in z relative to $\overline{z}$ captures this.

We simulate the model for different values of z while holding $\alpha = \overline{\alpha}$. The results are reported in the second column of Table 4. The estimated value of σ_z in Panel A is statistically significant. However, the overall fit as measured by SSR is much worse than the model with only variations in α, and three of the four correlations predicted in Panel B are negative, in contrast to the data. As z increases, investing in the long-term project becomes more attractive. The price-setter k thus increases ξ_k to boost its funding share x_k and invest more. However, as x_k increases, the aggregate feasibility constraint in state B tightens, necessitating higher $E(r)$ so that the price-takers bring enough liquidity to clear the market. The higher cost of liquidity $E(r)$ offsets the incentive of the price-takers to shift more funding into shadow banking, where liquidity regulation does not constrain how much they can invest. The net effect on ξ_j is negative, producing the wrong correlations relative to the data.

5.4.3 Money Supply Shocks

Money supply shocks are introduced by allowing for exogenous variation in external liquidity:

$$L = \overline{L} + \varepsilon_L$$

where ε_L is normally distributed with mean 0 and variance σ_L^2. We simulate the model for different draws of ε_L while holding $\alpha = \overline{\alpha}$ and $z = \overline{z}$.

The results are reported in the third column of Table 4. The overall fit is better than the model with only shocks to z but still substantially worse than the model with only shocks to α. As was the case with loan demand shocks, several of the correlations predicted in Panel B are negative, in contrast to the data. All else constant, a decrease in external liquidity increases $E(r)$ but reduces both ξ_j and ξ_k. The decrease in ξ_j reflects the fact that price-takers have less of a regulatory arbitrage motive when the expected interbank rate is high, and the decrease in ξ_k reflects the fact that the price-setter is competing against less aggressive products by the price-takers. Money supply shocks thus generate negative correlations between the interbank rate and WMP returns, contradicting the positive correlations in the data.

5.4.4 Multiple, Simultaneous Shocks

Now consider a version of the quantitative model which has shocks to liquidity regulation, shocks to loan demand, and money supply shocks, all at the same time. The shocks (ε_α, ε_z, and ε_L) are drawn from the relevant distributions, all of which are assumed to be independent of each other. We are able to separately identify σ_α, σ_z, and σ_L since shocks to liquidity regulation, loan demand, and external liquidity imply different correlations between $E(r)$, ξ_j, and ξ_k, as discussed above.

The results are reported in the fourth column of Table 4. The quantitative model with three shocks matches the four empirical correlations very well. The SSR drops to 0.02, two orders of magnitude smaller than that in any of the models with only one shock. Moreover, σ_α, σ_z, and σ_L are all statistically significant, indicating that all three shocks are relevant. However, as we saw when we considered each shock separately, shocks to liquidity regulation play a much more important role than shocks to either loan demand or external liquidity when it comes to getting the right signs for the correlations.

To this point, we also find that variations in α explain 23%, 99%, and 90% of the variance of $E(r)$, ξ_j, and ξ_k in the estimated model, respectively. Variations in z explain 64% of the variance of $E(r)$, indicating that loan demand shocks were important for the variance of the interbank rate. This complements our finding in Section 5.3 that changes in liquidity regulation can explain about one-third of the increase in the interbank repo rate between 2007 and 2014, along with explaining 60% of the increase in the aggregate credit-to-savings ratio.

6 Conclusion

This paper has developed a theoretical framework to study the endogenous response of the banking sector to liquidity regulation and the implications for the aggregate economy. We showed that the introduction of a liquidity floor can generate an unintended credit boom when there is interbank market power. Liquidity floors are endogenously more binding on an interbank price-taker than on an interbank price-setter. In response, the price-takers find it optimal to offer a new savings instrument and manage the funds raised by this instrument in an off-balance-sheet vehicle that is not subject to liquidity regulation. The push to attract savings into off-balance-sheet instruments raises the interest rates on these instruments above the rates on traditional deposits and poaches funding from the price-setter. The price-setter does not find it optimal to completely undo the reallocation of savings by offering equally high returns. Instead, the price-setter may engage in a form of asymmetric competition, using its market power to tighten the interbank market for emergency liquidity against the price-takers. The new equilibrium is characterized by more credit as savings are reallocated across banks and lending is reallocated across markets.

Applying our framework to China, where the interbank market is characterized by market power, we found that a regulatory push to increase bank liquidity and cap loan-to-deposit ratios in the late 2000s accounts for over half of China's unprecedented credit boom between 2007 and 2014. A quantitative extension that allowed for other, non-regulatory shocks also identified variation in liquidity rules as the dominant force behind observed co-movements in market-determined interest rates.

Our model also helps to understand the consequences of liquidity regulation in more general scenarios. The core ingredients of the model connect to features of the National Banking Era in the U.S., prior to the creation of the Federal Reserve. Impediments to interest rate targeting by central banks at the zero lower bound may also re-ignite market power in funding markets between banks. Our model suggests that optimal regulation in these environments cannot take the form of a simple liquidity floor. It is the combination of interbank market power and shadow banking that is problematic, as such a floor was shown to be optimal otherwise. Further study of optimal policy in a quantitative model with interbank market power is an important avenue for future research.

References

Acharya, V., D. Gromb, and T. Yorulmazer. 2012. "Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking." *American Economic Journal: Macroeconomics*, 4(2), 184-217.

Acharya, V., P. Schnabl, and G. Suarez. 2013. "Securitization Without Risk Transfer." *Journal of Financial Economics*, 107(3), 515-536.

Adrian, T., A. Ashcraft, and N. Cetorelli. 2013. "Shadow Bank Monitoring." Staff Report No. 638, Federal Reserve Bank of New York.

Adrian, T. and N. Boyarchenko. 2018. "Liquidity Policies and Systemic Risk." *Journal of Financial Intermediation*, Special Issue on "Banking and Regulation: The Next Frontier" (eds. M. Dewatripont and D. Hancock), 45-60.

Afonso, G. and R. Lagos. 2015. "Trade Dynamics in the Market for Federal Funds." *Econometrica*, 83(1), 263-313.

Aldasoro, I. and E. Faia. 2016. "Systemic Loops and Liquidity Regulation." *Journal of Financial Stability*, 27(1), 1-16.

Allen, F., E. Carletti, and R. Marquez. 2015. "Deposits and Bank Capital Structure." *Journal of Financial Economics*, 118(3), 601-619.

Allen, F. and D. Gale. 2004. "Financial Intermediaries and Markets." *Econometrica*, 72(4), 1023-1061.

Allen, F. and D. Gale. 2017. "How Should Bank Liquidity be Regulated?" In *Achieving Financial Stability: Challenges to Prudential Regulation* (eds. D. Evanoff, G. Kaufman, A. Leonello, and S. Manganelli), World Scientific, 135-157.

Ashcraft, A. and D. Duffie. 2007. "Systemic Illiquidity in the Federal Funds Market." *American Economic Review*, 97(2), 221-225.

Bai, C., C. Hsieh, and Z. Song. 2016. "The Long Shadow of a Fiscal Expansion." *Brookings Papers on Economic Activity*, Fall 2016, 129-165.

Banerjee, R. and H. Mio. 2018. "The Impact of Liquidity Regulation on Banks." *Journal of Financial Intermediation*, Special Issue on "Banking and Regulation: The Next Frontier" (eds. M. Dewatripont and D. Hancock), 30-44.

Begenau, J. 2020. "Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model." *Journal of Financial Economics*, 136(2), 355-378.

Begenau, J. and T. Landvoigt. 2018. "Financial Regulation in a Quantitative Model of the Modern Banking System." Manuscript, Stanford University.

Bhattacharya, S. and D. Gale. 1987. "Preference Shocks, Liquidity, and Central Bank Policy." In *New Approaches to Monetary Economics* (eds. W. Barnett and K. Singleton), Cambridge University Press, 69-88.

Buchak, G., G. Matvos, T. Piskorski, and A. Seru. 2018. "Fintech, Regulatory Arbitrage, and the Rise of Shadow Banks." *Journal of Financial Economics*, 130(3), 453-483.

Chen, K., J. Ren, and T. Zha. 2018. "The Nexus of Monetary Policy and Shadow Banking in China." *American Economic Review*, 108(12), 3891-3936.

Corbae, D. and P. D'Erasmo. 2013. "A Quantitative Model of Banking Industry Dynamics." Manuscript, University of Wisconsin-Madison.

Corbae, D. and P. D'Erasmo. 2019. "Capital Requirements in a Quantitative Model of Banking Industry Dynamics." Working Paper 25424, NBER.

Craig, B. and G. von Peter. 2014. "Interbank Tiering and Money Center Banks." *Journal of Financial Intermediation*, 23(3), 322-347.

Dávila, E. and A. Walther. 2020. "Does Size Matter? Bailouts with Large and Small Banks." *Journal of Financial Economics*, 136(1), 1-22.

Davis, D., O. Korenok, J. Lightle, and E.S. Prescott. 2020. "Liquidity Requirements and the Interbank Loan Market: An Experimental Investigation." *Journal of Monetary Economics*, 115 (November), 113-126..

Demyanyk, Y. and E. Loutskina. 2016. "Mortgage Companies and Regulatory Arbitrage." *Journal of Financial Economics*, 122(2), 328-351.

Diamond, D. and P. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3), 401-419.

Diamond, D. and A. Kashyap. 2016. "Liquidity Requirements, Liquidity Choice, and Financial Stability." In *Handbook of Macroeconomics, Volume 2* (eds. J. Taylor and H. Uhlig), 2263-2303.

Dobson, W. and A. Kashyap. 2006. "The Contradiction in China's Gradualist Banking Reforms." *Brookings Papers on Economic Activity*, Fall 2006, 103-148.

Drechsler, I., A. Savov, and P. Schnabl. 2017. "The Deposits Channel of Monetary Policy." *Quarterly Journal of Economics*, 132(4), 1819-1876.

Duffie, D., N. Gârleanu, and L. Pedersen. 2005. "Over-the-Counter Markets." *Econometrica*, 73(6), 1815-1847.

Egan, M., A. Hortaçsu, and G. Matvos. 2017. "Deposit Competition and Financial Fragility: Evidence from the US Banking Sector." *American Economic Review*, 107(1), 169-216.

Elliott, D., A. Kroeber, and Y. Qiao. 2015. "Shadow Banking in China: A Primer." Report, Brookings, April 1, 2015.

Farhi, E., M. Golosov, and A. Tsyvinski. 2007. "A Theory of Liquidity and Regulation of Financial Intermediation." Working Paper 12959, NBER.

Farhi, E., M. Golosov, and A. Tsyvinski. 2009. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76(3), 973-992.

Frydman, C., E. Hilt, and L. Zhou. 2015. "Economic Effects of Early Shadow Banks: Trust Companies and the Panic of 1907." *Journal of Political Economy*, 123(4), 902-940.

Gennaioli, N. and A. Shleifer, 2010, "What Comes to Mind," *Quarterly Journal of Economics*, 125(4), 1399-1433.

Gorton, G., T. Laarits, and T. Muir. 2020. "Mobile Collateral versus Immobile Collateral." Forthcoming in *Journal of Money, Credit and Banking.*

Gorton, G. and E. Tallman. 2016. "How Did Pre-Fed Banking Panics End?" Working Paper 22036, NBER.

Gu, X. and L. Yun. 2019. "The Unintended Consequences of Regulation: Evidence from China's Interbank Markets." Working Paper No. 06/2019, Hong Kong Institute for Monetary Research.

Hachem, K. 2018. "Shadow Banking in China." *Annual Review of Financial Economics*, 10(1), 287-308.

Harris, M., C. Opp, and M. Opp. 2014. "Higher Capital Requirements, Safer Banks? Macroprudential Regulation in a Competitive Financial System." Working Paper, SSRN.

Huang, J. 2018. "Banking and Shadow Banking." *Journal of Economic Theory*, 178 (November), 124-152.

Jin, H. and C. Xiong. 2018. "Financial Openness, Bank Capital Flows, and the Effectiveness of Macroprudential Policies." Working Paper 2018-007, Center for Applied Economics and Policy Research, Indiana University.

Keeley, M. 1990. "Deposit Insurance, Risk, and Market Power in Banking." *American Economic Review*, 80(5), 1183-1200.

Mitchener, K. and G. Richardson. 2019. "Network Contagion and Interbank Amplification During the Great Depression." *Journal of Political Economy*, 127(2), 465-507.

Moen, J. and E. Tallman. 2000. "Clearinghouse Membership and Deposit Contraction during the Panic of 1907." *Journal of Economic History*, 60(1), 145-163.

Moen, J. and E. Tallman. 2014. "Outside Lending in the NYC Call Loan Market." Working Paper 14-08, Federal Reserve Bank of Cleveland.

Neumark, D. and S. Sharpe. 1992. "Market Structure and the Nature of Price Rigidity: Evidence from the Market for Consumer Deposits." *Quarterly Journal of Economics*, 107(2), 657-680.

Plantin, G. 2015. "Shadow Banking and Bank Capital Regulation." *Review of Financial Studies*, 28(1), 146-175.

Robatto, R. 2019. "Liquidity Requirements and Central Bank Interventions during Banking Crises." Manuscript, University of Wisconsin-Madison.

Song, Z. and W. Xiong. 2018. "Risks in China's Financial System." *Annual Review of Financial Economics*, 10(1), 261-286.

Sprague, O. 1910. *History of Crises under the National Banking System*. National Monetary Commission. Washington: Government Printing Office.

Van den Heuvel, S. 2018. "The Welfare Effects of Bank Liquidity and Capital Requirements." Manuscript, Federal Reserve Board.

Table 1

Interbank Repo Rate Regressions

	(1)	(2)
	repo	repo
L1.repo	0.827***	0.826***
	(0.0377)	(0.0377)
L2.repo	0.0693*	0.0618
	(0.0383)	(0.0384)
RRR	0.209***	0.202***
	(0.0713)	(0.0711)
L.NCD_Big4	0.0394***	
	(0.0120)	
L.NCD_ABC		0.0227*
		(0.0118)
L.NCD_BOC		-0.0210
		(0.0161)
L.NCD_CCB		0.0414***
		(0.0122)
L.NCD_ICBC		-0.0135
		(0.0475)
Observations	748	748
R-squared	0.842	0.844

Notes: Dependent variable is the repo rate on day t; L1.repo and L2.repo are one- and two-day lags respectively; RRR is the required reserve ratio; L.NCD is a dummy variable for NCD announcement by the indicated bank(s) on day $t-1$. Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 2

Calibration Results

	(1) Model $\alpha = 0.145$	(2) Data 2007	(3) Model $\alpha = 0.25$	(4) Data 2014
Average Interbank Rate, $E(r)$	0.2%	0.2%	0.6%	1.3%
Price-Setter Loan-to-Deposit Ratio, $1-\lambda_k$	0.62	0.62	0.71	0.70
Credit-to-Savings Ratio, $1-\lambda_j-(\lambda_k-\lambda_j)\,x_k$	72.5%	72.5%	78.7%	82.5%

Notes: We target the 2007 values of all variables in this table. The 2014 values in (3) are generated by the calibrated model keeping all parameters except α unchanged. Interbank rates in (2) and (4) are reported net of the benchmark deposit rate.

Table 3

Correlations Between Market Rates in the Data

$corr\left(E\left(r\right),\xi_j\right)$	0.456
	(0.077)
$corr\left(E\left(r\right),\xi_k\right)$	0.329
	(0.095)
$corr\left(E\left(r\right),\xi_j-\xi_k\right)$	0.259
	(0.093)
$corr\left(\xi_j,\xi_k\right)$	0.736
	(0.052)

Notes: Bootstrapped standard errors are in parentheses.

Table 4

Estimation Results

	Panel A: Parameter Values				
	Model with only σ_α	Model with only σ_z	Model with only σ_L	Model with σ_α, σ_z, σ_L	
σ_α	0.0131	-	-	0.0396	
	(0.0001)			(0.0030)	
σ_z	-	0.0037	-	0.0002	
		(0.0004)		(0.0000)	
σ_L	-	-	0.0059	0.0014	
			(0.0002)	(0.0007)	
SSR	1.052	6.382	3.886	0.021	
	Panel B: Pairwise Correlations				
	Model with only σ_α	Model with only σ_z	Model with only σ_L	Model with σ_α, σ_z, σ_L	Data
$corr\left(E\left(r\right), \xi_j\right)$	0.955	-0.999	-1.000	0.371	0.456
$corr\left(E\left(r\right), \xi_k\right)$	0.784	0.861	-0.960	0.352	0.329
$corr\left(E\left(r\right), \xi_j - \xi_k\right)$	0.994	-0.959	0.568	0.296	0.259
$corr\left(\xi_j, \xi_k\right)$	0.931	-0.849	0.960	0.846	0.736

Notes: Panel A reports the estimated parameter values. SSR is the sum of squared residuals. Bootstrapped standard errors are in parentheses. Columns 1 to 4 in Panel B report the simulated correlations using the estimated parameter values in each model. Column 5 in Panel B reports the correlations in the data as per Table 3.

Online Appendix for

"Liquidity Rules and Credit Booms"

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Appendix A – Proofs

Proof of Proposition 1

Letting $\eta_s \geq 0$ and $\upsilon_s \geq 0$ denote Lagrange multipliers, the Lagrangian for k's problem is

$$\begin{aligned}\mathcal{L}_k &= \left[g\left(1-\lambda_k\right)+\lambda_k-1\right]x_k^0+\left[\widetilde{\pi}r_A\left(\theta_j^A-\lambda_j\right)+\left(1-\widetilde{\pi}\right)r_B\left(\theta_j^B-\lambda_j\right)\right]\left(1-x_k^0\right)\\ &\quad +\eta_A\left(\overline{r}_A-r_A\right)+\eta_B r_B+\sum_{s\in\{A,B\}}\upsilon_s\left[\left(\lambda_j-\theta_j^s\right)\left(1-x_k^0\right)+\left(\lambda_k-\theta_k^s\right)x_k^0\right]\end{aligned}$$

We ignore for the moment the constraints $r_A \geq 0$ and $r_B \leq \overline{r}_B$; we will solve the relaxed problem without these constraints and then return to them.

The FOCs for λ_k, r_A, and r_B are respectively:

$$0=\frac{\partial\mathcal{L}_k}{\partial\lambda_k}=\left(-g'\left(1-\lambda_k\right)+1+\sum_{s\in\{A,B\}}\upsilon_s\right)x_k^0 \tag{A.1}$$

$$0=\frac{\partial\mathcal{L}_k}{\partial r_A}=\widetilde{\pi}\left[\theta_j^A-\lambda_j+\left(\sum_{s\in\{A,B\}}\upsilon_s-E\left(r\right)\right)\frac{\partial\lambda_j}{\partial E\left(r\right)}\right]\left(1-x_k^0\right)-\eta_A \tag{A.2}$$

$$0=\frac{\partial\mathcal{L}_k}{\partial r_B}=\left(1-\widetilde{\pi}\right)\left[\theta_j^B-\lambda_j+\left(\sum_{s\in\{A,B\}}\upsilon_s-E\left(r\right)\right)\frac{\partial\lambda_j}{\partial E\left(r\right)}\right]\left(1-x_k^0\right)+\eta_B \tag{A.3}$$

where

$$\frac{\partial\lambda_j}{\partial E\left(r\right)}=\frac{1}{-g''\left(1-\lambda_j\right)}$$

from Eq. (3).

The properties $g'(1) > 1$ and $g''(\cdot) < 0$ imply $g'(\cdot) > 1$ and thus $\upsilon_A+\upsilon_B > 0$ from Eq. (A.1), i.e., aggregate feasibility holds with equality in at least one state, as per Lemma 1. λ_k is then pinned down by Eq. (5), which can be rearranged to get

$$\lambda_k=\theta_k^{s'}-\left(\lambda_j-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0} \tag{A.4}$$

Next, use Eqs. (3) and (A.1) to rewrite Eqs. (A.2) and (A.3) as

$$\eta_A=\frac{\widetilde{\pi}\left(1-x_k^0\right)}{-g''\left(1-\lambda_j\right)}\left[g'\left(1-\lambda_k\right)-g'\left(1-\lambda_j\right)-g''\left(1-\lambda_j\right)\left(\theta_j^A-\lambda_j\right)\right] \tag{A.5}$$

$$\eta_B=-\frac{\left(1-\widetilde{\pi}\right)\left(1-x_k^0\right)}{-g''\left(1-\lambda_j\right)}\left[g'\left(1-\lambda_k\right)-g'\left(1-\lambda_j\right)+g''\left(1-\lambda_j\right)\left(\lambda_j-\theta_j^B\right)\right] \tag{A.6}$$

If $r_B > 0$, then $\eta_B = 0$ by complementary slackness, so Eq. (A.6) reduces to

$$g'(1-\lambda_k) - g'(1-\lambda_j) + g''(1-\lambda_j)\left(\lambda_j - \theta_j^B\right) = 0 \tag{A.7}$$

and we can rewrite Eq. (A.5) as

$$\eta_A = \widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)\left(1 - x_k^0\right) > 0 \tag{A.8}$$

which implies $r_A = \overline{r}_A$ by complementary slackness. The constraint $r_A \geq 0$ is then trivially satisfied by $\overline{r}_A \geq 0$.

Since $g''(\cdot) < 0$, it follows immediately from Eq. (A.7) that $\lambda_k > \lambda_j$ in any equilibrium where $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$.

Now return to $r_B \leq \overline{r}_B$. Let $\left(\widehat{r}_B, \widehat{\lambda}_k, \widehat{\lambda}_j\right)$ denote the solution to the relaxed problem above and $\left(r_B^*, \lambda_k^*, \lambda_j^*\right)$ the solution to the true problem where k is also subject to $r_B \leq \overline{r}_B$. If $\widehat{r}_B \leq \overline{r}_B$, then the two solutions coincide and $\lambda_k^* > \lambda_j^*$ follows from $\widehat{\lambda}_k > \widehat{\lambda}_j$ as shown earlier. If instead $\widehat{r}_B > \overline{r}_B$, then $r_B^* = \overline{r}_B$ and thus $r_B^* < \widehat{r}_B$. Moreover, $\eta_A > 0$ and Eq. (A.4) still hold, so we conclude $\lambda_j^* < \widehat{\lambda}_j$ from $r_B^* < \widehat{r}_B$ and Eq. (3) followed by $\lambda_k^* > \widehat{\lambda}_k$ from Eq. (A.4). The result $\lambda_k^* > \lambda_j^*$ then follows from $\widehat{\lambda}_k > \widehat{\lambda}_j$. ■

Proof of Proposition 2

If $r_B > 0$, then the equilibrium value of λ_j solves Eq. (A.7) with λ_k and η_A as per Eqs. (A.4) and (A.8). Formally, this pins down λ_j as the solution to

$$G_1\left(\lambda_j; x_k^0\right) = 0 \tag{A.9}$$

where

$$G_1\left(\lambda_j; x_k^0\right) \equiv g'\left(1 - \theta_k^{s'} + \left(\lambda_j - \theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right) - g'(1-\lambda_j) + g''(1-\lambda_j)\left(\lambda_j - \theta_j^B\right)$$

Notice $\frac{\partial G_1\left(\lambda_j; x_k^0\right)}{\partial \lambda_j} < 0$ for $\lambda_j \geq \theta_j^B$ by the properties $g''(\cdot) < 0$ and $g'''(\cdot) \geq 0$. Therefore, if Eq. (A.9) has a solution $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$, the solution is unique. Moreover, we can establish $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ by establishing $G_1\left(\theta_j^B; x_k^0\right) > 0$ and $G_1\left(\theta_j^A; x_k^0\right) < 0$ then invoking the intermediate value theorem.

Start with $G_1\left(\theta_j^B; x_k^0\right) > 0$. If $s' = A$, then $G_1\left(\theta_j^B; x_k^0\right) > 0$ is equivalent to

$$g'\left(1 - \theta_k^A - \left(\theta_j^A - \theta_j^B\right)\frac{1-x_k^0}{x_k^0}\right) - g'\left(1 - \theta_j^B\right) > 0$$

which, with $g''(\cdot) < 0$, requires

$$1-\theta_k^A-\left(\theta_j^A-\theta_j^B\right)\frac{1-x_k^0}{x_k^0}<1-\theta_j^B$$

or equivalently

$$x_k^0<\frac{\theta_j^A-\theta_j^B}{\theta_j^A-\theta_k^A}$$

Note that $s'=A$ implies $\theta_j^A\left(1-x_k^0\right)+\theta_k^A x_k^0\geq\theta_j^B\left(1-x_k^0\right)+\theta_k^B x_k^0$, which rearranges to $x_k^0\leq\frac{\theta_j^A-\theta_j^B}{\theta_j^A-\theta_k^A+\theta_k^B-\theta_j^B}$ so the condition for $G_1\left(\theta_j^B;x_k^0\right)>0$ is true. If instead $s'=B$, then $G_1\left(\theta_j^B;x_k^0\right)>0$ is equivalent to

$$g'\left(1-\theta_k^B\right)-g'\left(1-\theta_j^B\right)>0$$

which is also true by $\theta_k^B>\theta_j^B$ and $g''\left(\cdot\right)<0$.

Consider next $G_1\left(\theta_j^A;x_k^0\right)<0$. If $s'=A$, then $G_1\left(\theta_j^A;x_k^0\right)<0$ is equivalent to

$$g'\left(1-\theta_k^A\right)-g'\left(1-\theta_j^A\right)+g''\left(1-\theta_j^A\right)\left(\theta_j^A-\theta_j^B\right)<0$$

which is true by $\theta_j^A>\max\left\{\theta_k^A,\theta_j^B\right\}$ and $g''\left(\cdot\right)<0$. If instead $s'=B$, then $G_1\left(\theta_j^A;x_k^0\right)<0$ is equivalent to

$$g'\left(1-\theta_k^B+\left(\theta_j^A-\theta_j^B\right)\frac{1-x_k^0}{x_k^0}\right)-g'\left(1-\theta_j^A\right)+g''\left(1-\theta_j^A\right)\left(\theta_j^A-\theta_j^B\right)<0$$

which will be true for θ_j^A sufficiently high (e.g., it is trivially true for $\theta_j^A\geq\theta_j^B+\left(\theta_k^B-\theta_j^B\right)x_k^0$ and also true for some lower θ_j^A on account of $g''\left(\cdot\right)<0$).

Finally, we find conditions under which the equilibrium has $r_B>0$. We recall from the proof of Proposition 1 that $r_A=\overline{r}_A$ if $r_B>0$, so, from Eq. (3), $r_B>0$ is confirmed if and only if

$$1+\widetilde{\pi}\overline{r}_A<g'\left(1-\lambda_j\right)\tag{A.10}$$

where λ_j solves Eq. (A.9). Notice that $\overline{r}_A$ does not enter Eq. (A.9). Also recall $g'\left(\cdot\right)>1$. Therefore, Condition (A.10) defines a positive upper bound on $\overline{r}_A$ such that, for any $\overline{r}_A$ below this upper bound, the equilibrium has $r_B>0$. ■

Proof of Lemma 2

The Lagrange function for bank j's problem is:

$$\mathcal{L}_j=g\left(1-\lambda_j\right)+\lambda_j-1-\xi_j+\widetilde{\pi}r_A\left(\lambda_j-\theta_j^A\right)+\left(1-\widetilde{\pi}\right)r_B\left(\lambda_j-\theta_j^B\right)+\mu_j\left[\lambda_j-\alpha\left(1-h\left(\xi_j\right)\right)\right]+\rho_j\xi_j$$

where $\mu_j \geq 0$ and $\rho_j \geq 0$ are the multipliers on the constraints. The FOCs for λ_j and ξ_j are then given by Eqs. (8) and (9), with the complementary slackness conditions (10) and (11).

There are three cases:

1. The first case is $\mu_j = 0$. Eq. (9) delivers $\rho_j = 1$ and hence $\xi_j = 0$ by complementary slackness. Eq. (8) collapses to Eq. (3) from the unregulated equilibrium and hence defines the function $\lambda_j(E(r))$ with $\lambda_j'(E(r)) > 0$. Confirming $\mu_j = 0$ requires $\lambda_j \geq \alpha$, i.e.,
$$E(r) \geq g'(1-\alpha) - 1 \equiv \overline{R}(\alpha)$$
2. The second case is $\mu_j > 0$ and $\rho_j = 0$. By complementary slackness,
$$\lambda_j = \alpha\left(1 - h\left(\xi_j\right)\right)$$
which combines with Eqs. (8) and (9) to isolate ξ_j as
$$g'\left(1 - \alpha\left(1 - h\left(\xi_j\right)\right)\right) - \frac{1}{\alpha h'\left(\xi_j\right)} = 1 + E(r) \qquad \text{(A.11)}$$
The left-hand side of Eq. (A.11) is decreasing in ξ_j, so Eq. (A.11) defines a function $\xi_j(E(r))$ with $\xi_j'(E(r)) < 0$. Confirming $\xi_j \geq 0$ requires
$$E(r) \leq g'(1-\alpha) - 1 - \frac{1}{\alpha h'(0)} \equiv \underline{R}(\alpha)$$
3. The third case is $\mu_j > 0$ and $\rho_j > 0$. By complementary slackness, $\lambda_j = \alpha$ and $\xi_j = 0$. Eqs. (8) and (9) then become
$$\mu_j = g'(1-\alpha) - 1 - E(r)$$
$$\rho_j = 1 - \alpha h'(0)\,\mu_j$$
Confirming $\mu_j > 0$ and $\rho_j > 0$ requires $E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right)$.

Putting together the cases, the optimality conditions from j's problem are
$$\lambda_j : \begin{cases} \lambda_j = \alpha\left(1 - h\left(\xi_j\right)\right) & if \quad E(r) \leq \underline{R}(\alpha) \\ \lambda_j = \alpha & if \quad E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right) \\ g'(1-\lambda_j) = 1 + E(r) & if \quad E(r) \geq \overline{R}(\alpha) \end{cases} \qquad \text{(A.12)}$$

with

$$\xi_j : \begin{cases} g'\left(1-\alpha\left(1-h\left(\xi_j\right)\right)\right) - \frac{1}{\alpha h'\left(\xi_j\right)} = 1 + E\left(r\right) & if \quad E\left(r\right) \leq \underline{R}\left(\alpha\right) \\ \xi_j = 0 & if \quad E\left(r\right) > \underline{R}\left(\alpha\right) \end{cases} \tag{A.13}$$

where $\frac{\partial \xi_j}{\partial E(r)} < 0$ for $E\left(r\right) \leq \underline{R}\left(\alpha\right)$ follows from the properties of $g\left(\cdot\right)$ and $h\left(\cdot\right)$. ■

Proof of Lemma 3

We first set up k's problem without restricting $\xi_k = 0$. The Lagrangian is:

$$\begin{aligned} \mathcal{L}_k \;=\; & \left[g\left(1-\lambda_k\right) + \lambda_k - 1 - \xi_k\right] x_k + \left[\widetilde{\pi}\overline{r}_A\left(\theta_j^A - \theta_j^B\right) + E\left(r\right)\left(\theta_j^B - \lambda_j\right)\right]\left(1-x_k\right) \\ & + \mu_k\left[\lambda_k - \alpha\left(1-h\left(\xi_k\right)\right)\right] x_k + \rho_k \xi_k x_k + \sum_{s\in\{A,B\}} \upsilon_s\left[\left(\lambda_j - \theta_j^s\right)\left(1-x_k\right) + \left(\lambda_k - \theta_k^s\right) x_k\right] \end{aligned}$$

where λ_j, ξ_j, and x_k are given by Eqs. (A.12), (A.13), and (7) respectively. Here, we are assuming the choice of $E\left(r\right)$ will satisfy $r_B > 0$; we verify this later in the proof. To simplify the exposition, we explicitly assume $g\left(\cdot\right)$ and $h\left(\cdot\right)$ such that k's problem is concave on the interval $E\left(r\right) \leq \underline{R}\left(\alpha\right)$.

The FOCs with respect to λ_k and ξ_k are:

$$\frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \left(-g'\left(1-\lambda_k\right) + 1 + \upsilon_{s'} + \mu_k\right) x_k \tag{A.14}$$

$$\frac{\partial \mathcal{L}_k}{\partial \xi_k} = \left[\alpha \mu_k h'\left(\xi_k\right) + \rho_k - 1\right] x_k + \delta_1 Z \tag{A.15}$$

where $\upsilon_{s'} \equiv \upsilon_A + \upsilon_B$ and Z is an endogenous object defined as

$$Z \equiv g\left(1-\lambda_k\right) + \lambda_k - 1 - \xi_k - \widetilde{\pi}\overline{r}_A\left(\theta_j^A - \theta_j^B\right) + E\left(r\right)\left(\lambda_j - \theta_j^B\right) - \frac{\upsilon_{s'}\left(\lambda_j - \theta_j^{s'}\right)}{x_k}$$

Turning to the choice of $E\left(r\right)$, we notice that $\mathcal{L}_k$ is continuous but not continuously differentiable because λ_j and ξ_j have kinks at the cutoffs $\underline{R}\left(\alpha\right)$ and $\overline{R}\left(\alpha\right)$. For values of $E\left(r\right)$ where $\mathcal{L}_k$ is differentiable:

$$\frac{\partial \mathcal{L}_k}{\partial E\left(r\right)} = \left[\theta_j^B - \lambda_j + \left(\upsilon_{s'} - E\left(r\right)\right)\frac{\partial \lambda_j}{\partial E\left(r\right)}\right]\left(1-x_k\right) - \delta_1 Z \frac{\partial \xi_j}{\partial E\left(r\right)} \tag{A.16}$$

where $\frac{\partial \lambda_j}{\partial E(r)}$ and $\frac{\partial \xi_j}{\partial E(r)}$ are governed by Eqs. (A.12) and (A.13).

If $\mu_k = 0$, then setting $\frac{\partial \mathcal{L}_k}{\partial \lambda_k} = 0$ in Eq. (A.14) delivers

$$\upsilon_{s'} = g'\left(1-\lambda_k\right) - 1 \tag{A.17}$$

which is the simplified form of Eq. (A.1) and, by the properties of $g'(\cdot)$, requires $\upsilon_{s'} > 0$. In other words, aggregate feasibility binds in state s' so we can use

$$\lambda_k = \theta_k^{s'} - \left(\lambda_j - \theta_j^{s'}\right) \frac{1-x_k}{x_k} \tag{A.18}$$

in the derivations that follow. Eq. (A.18) is the same as Eq. (A.4) but evaluated at the more general x_k in Eq. (7).

With $h'(0) \to \infty$, there is only one cutoff $\overline{R}(\alpha)$ in Eqs. (A.12) and (A.13). Moreover, $\xi_k = 0$ implies $\mu_k = 0$, otherwise $\frac{\partial \mathcal{L}_k}{\partial \xi_k} = \infty$ in Eq. (A.15) and $\xi_k = 0$ cannot be optimal. We fix $\xi_k = 0$ as per the statement of the proposition for the rest of this proof.

For $E(r) \geq \overline{R}(\alpha)$, Eqs. (A.12) and (A.13) imply $\xi_j = 0$ and

$$\frac{\partial \lambda_j}{\partial E(r)} = -\left(g''(1-\lambda_j)\right)^{-1}$$

so Eq. (A.16) becomes

$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} G_1\left(\lambda_j; x_k^0\right)$$

where $G_1(\cdot)$ is as defined in the proof of Proposition 2 and λ_j^* is the only solution to $G_1\left(\lambda_j; x_k^0\right) = 0$ on the interval $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$. From Eq. (3), the interbank rate is then $E(r) = g'\left(1-\lambda_j^*\right) - 1$, so $E(r) > \overline{R}(\alpha)$ if and only if $\alpha < \lambda_j^*$. Corollarily, $\overline{R}(\alpha)$ dominates any $E(r) > \overline{R}(\alpha)$ if $\alpha \geq \lambda_j^*$. In other words, the equilibrium for $\alpha \geq \lambda_j^*$ will have $E(r) \leq \overline{R}(\alpha)$, which, from the proof of Lemma 2, means $\lambda_j = \alpha\left(1 - h\left(\xi_j\right)\right)$.

For $E(r) \leq \overline{R}(\alpha)$, Eqs. (A.12) and (A.13) imply

$$\frac{\partial \lambda_j}{\partial E(r)} = -\alpha h'\left(\xi_j\right) \frac{\partial \xi_j}{\partial E(r)}$$

and

$$\frac{\partial \xi_j}{\partial E(r)} = \left(\alpha h'\left(\xi_j\right) g''\left(1 - \alpha\left(1 - h\left(\xi_j\right)\right)\right) + \frac{h''\left(\xi_j\right)}{\alpha\left(h'\left(\xi_j\right)\right)^2}\right)^{-1}$$

so Eq. (A.16) becomes

$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} \widetilde{G}\left(\xi_j; \alpha\right)$$

where we define

$$\begin{aligned} \widetilde{G}\left(\xi_j; \alpha\right) \equiv\ & G_1\left(\alpha\left(1-h\left(\xi_j\right)\right); x_k^0 - \delta_1 \xi_j\right) + \frac{1}{\alpha h'\left(\xi_j\right)} + \frac{h''\left(\xi_j\right)}{\alpha^2\left(h'\left(\xi_j\right)\right)^3}\left[\alpha\left(1-h\left(\xi_j\right)\right) - \theta_j^B\right] \\ & + \frac{\frac{\delta_1}{\alpha h'\left(\xi_j\right)}\left[F\left(\alpha\left(1-h\left(\xi_j\right)\right), x_k^0 - \delta_1 \xi_j\right) - \frac{\alpha\left(1-h\left(\xi_j\right)\right) - \theta_j^B}{\alpha h'\left(\xi_j\right)}\right]}{1 - x_k^0 + \delta_1 \xi_j} \end{aligned} \tag{A.19}$$

and

$$F(\lambda_j, x_k) \equiv g\left(1-\theta_k^{s'}+\left(\lambda_j-\theta_j^{s'}\right)\frac{1-x_k}{x_k}\right)-\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)-\left(1-\theta_k^{s'}\right)-\left(\theta_j^{s'}-\theta_j^B\right)$$
$$+g'(1-\lambda_j)\left(\lambda_j-\theta_j^B\right)-g'\left(1-\theta_k^{s'}+\left(\lambda_j-\theta_j^{s'}\right)\frac{1-x_k}{x_k}\right)\frac{\lambda_j-\theta_j^{s'}}{x_k}$$

for future reference. Taking limits:

$$\lim_{E(r)\to\overline{R}(\alpha)^-}\frac{\partial\mathcal{L}_k}{\partial E(r)}\overset{sign}{=}G_1\left(\alpha;x_k^0\right)+\frac{1}{\alpha h'(0)}+\frac{h''(0)}{\alpha^2\left(h'(0)\right)^3}\left(\alpha-\theta_j^B\right)+\frac{\frac{\delta_1}{\alpha h'(0)}\left[F\left(\alpha,x_k^0\right)-\frac{\alpha-\theta_j^B}{\alpha h'(0)}\right]}{1-x_k^0}$$

so, with $h'(0)\to\infty$ and $\frac{h''(0)}{(h'(0))^3}\to 0$,

$$\lim_{E(r)\to\overline{R}(\alpha)^-}\frac{\partial\mathcal{L}_k}{\partial E(r)}\overset{sign}{=}G_1\left(\alpha;x_k^0\right)\leq G_1\left(\lambda_j^*;x_k^0\right)=0 \tag{A.20}$$

for $\alpha\geq\lambda_j^*$, where the inequality in Eq. (A.20) follows from the fact that $\frac{\partial G_1(\lambda_j;0)}{\partial\lambda_j}<0$ for $\lambda_j\geq\theta_j^B$; see again the proof of Proposition 2. The inequality in Eq. (A.20) holds strictly if and only if $\alpha>\lambda_j^*$, i.e., $E(r)<\overline{R}(\alpha)$ dominates $\overline{R}(\alpha)$ as α is pushed above λ_j^*. At $\alpha=\lambda_j^*$, the concavity of k's problem ensures that the only solution is $E(r)=\overline{R}\left(\lambda_j^*\right)$, which recovers the unregulated equilibrium.

Thus far, we have abstracted from the constraint $r_B>0$, which is equivalent to $E(r)>\widetilde{\pi}\overline{r}_A$ from the definition of $E(r)$. For $\alpha\geq\lambda_j^*$, Eq. (A.11) pins down $E(r)$ conditional on ξ_j and, with $\xi_k=0$, ξ_j is pinned down by $\widetilde{G}\left(\xi_j;\alpha\right)=0$. With $g(\cdot)$ and $h(\cdot)$ well-behaved, $\widetilde{G}\left(\xi_j;\alpha\right)=0$ defines ξ_j as a continuous function of α, hence $E(r)$ is also continuous in α for $\alpha\geq\lambda_j^*$ and the properties of the unregulated equilibrium (in this case $r_B>0$) carry over to the local analysis we do around $\alpha=\lambda_j^*$.[1] ■

Proof of Lemma 4

Consider k's problem without the constraint $\lambda_k\geq\alpha(1-h(\xi_k))$, i.e., set $\mu_k=0$. We will verify afterwards that the solution obtained without this constraint does indeed satisfy $\lambda_k\geq\alpha(1-h(\xi_k))$.

If $\xi_k>0$, then $\rho_k=0$ and we set $\frac{\partial\mathcal{L}_k}{\partial\xi_k}=0$ to get

$$\delta_1 Z=x_k \tag{A.21}$$

[1] The same line of argument can be used to conclude $r_B<\overline{r}_B$ for parameters that deliver this property in the unregulated equilibrium, i.e., $1+\widetilde{\pi}\overline{r}_A+(1-\widetilde{\pi})\overline{r}_B>g'(1-\lambda_j)$, where λ_j solves Eq. (A.9) independently of $\overline{r}_A$ and $\overline{r}_B$.

from Eq. (A.15), where

$$Z = F\left(\lambda_j, x_k\right) - \xi_k + \left(1 + E\left(r\right) - g'\left(1 - \lambda_j\right)\right)\left(\lambda_j - \theta_j^B\right) \tag{A.22}$$

and $F\left(\cdot\right)$ is as defined in the proof of Lemma 3. Substitute $\delta_1 Z = x_k$ into $\frac{\partial \mathcal{L}_k}{\partial E(r)} = 0$ to get

$$\frac{x_k}{1 - x_k}\frac{\partial \xi_j}{\partial E\left(r\right)} = \theta_j^B - \lambda_j + \left(g'\left(1 - \lambda_k\right) - 1 - E\left(r\right)\right)\frac{\partial \lambda_j}{\partial E\left(r\right)} \tag{A.23}$$

from Eq. (A.16).

If $E\left(r\right) \geq \overline{R}\left(\alpha\right)$, then $\xi_j = 0$ so Eq. (7) gives $x_k = x_k^0 + \delta_1 \xi_k$. Moreover, Eq. (A.22) simplifies to

$$Z = F\left(\lambda_j, x_k\right) - \xi_k$$

while Eq. (A.23) simplifies to $G_1\left(\lambda_j; x_k\right) = 0$, which is as defined in the proof of Proposition 2 and implicitly defines the function $\lambda_j\left(x_k\right)$. We can then rewrite Eq. (A.21) as

$$\xi_k = \frac{1}{2}\left(F\left(\lambda_j\left(x_k\right), x_k\right) - \frac{x_k^0}{\delta_1}\right) \tag{A.24}$$

Differentiate $F\left(\cdot\right)$ to find

$$\begin{aligned} &\frac{dF\left(\lambda_j\left(x_k\right), x_k\right)}{dx_k} \\ = \;& g''\left(1 - \theta_k^{s'} + \left(\lambda_j\left(x_k\right) - \theta_j^{s'}\right)\frac{1 - x_k}{x_k}\right)\left(1 - \frac{1 - x_k}{x_k}\frac{\lambda_j'\left(x_k\right)}{\frac{\lambda_j(x_k) - \theta_j^{s'}}{x_k^2}}\right)\frac{\left(\lambda_j\left(x_k\right) - \theta_j^{s'}\right)^2}{x_k^3} \\ & - \underbrace{G_1\left(\lambda_j\left(x_k\right); x_k\right)}_{=0}\lambda_j'\left(x_k\right) \end{aligned}$$

where

$$\lambda_j'\left(x_k\right) = \frac{1}{\frac{1 - x_k}{x_k} + \frac{2g''(1 - \lambda_j(x_k)) - g'''(1 - \lambda_j(x_k))\left(\lambda_j(x_k) - \theta_j^B\right)}{g''\left(1 - \theta_k^{s'} + \left(\lambda_j(x_k) - \theta_j^{s'}\right)\frac{1 - x_k}{x_k}\right)}}\frac{\lambda_j\left(x_k\right) - \theta_j^{s'}}{x_k^2}$$

from differentiation of $G_1\left(\cdot\right) = 0$. Therefore,

$$\frac{dF\left(\lambda_j\left(x_k\right), x_k\right)}{dx_k} < 0$$

so from Eq. (A.24)

$$\xi_k < \frac{1}{2}\left(F\left(\lambda_j^*, x_k^0\right) - \frac{x_k^0}{\delta_1}\right)$$

where $\lambda_j^* \Leftrightarrow \lambda_j\left(x_k^0\right)$ and $F\left(\lambda_j^*, x_k^0\right)$ is finite and independent of δ_1. If $F\left(\lambda_j^*, x_k^0\right) > 0$, which would be necessary for $\xi_k > 0$, then any $\delta_1 \in \left(0, \frac{x_k^0}{F\left(\lambda_j^*, x_k^0\right)}\right)$ will deliver $\xi_k < 0$ from Eq. (A.24), which contradicts $\xi_k > 0$. Therefore, $\xi_k = 0$ if $E(r) \geq \overline{R}(\alpha)$.

If instead $E(r) \leq \overline{R}(\alpha)$, then Lemma 2 and Eqs. (A.21), (A.22), and (A.23) imply

$$\xi_k = \frac{1}{2}\left(F\left(\alpha\left(1-h\left(\xi_j\right)\right), x_k^0 + \delta_1\left(\xi_k - \xi_j\right)\right) - \frac{\alpha\left(1-h\left(\xi_j\right)\right) - \theta_j^B}{\alpha h'\left(\xi_j\right)} + \xi_j - \frac{x_k^0}{\delta_1}\right) \quad \text{(A.25)}$$

where ξ_j solves[2]

$$G_1\left(\alpha\left(1-h\left(\xi_j\right)\right); x_k^0 + \delta_1\left(\xi_k - \xi_j\right)\right) + \frac{\frac{1}{\alpha h'\left(\xi_j\right)}}{1 - x_k^0 - \delta_1\left(\xi_k - \xi_j\right)} + \frac{h''\left(\xi_j\right)}{\alpha^2\left(h'\left(\xi_j\right)\right)^3}\left[\alpha\left(1-h\left(\xi_j\right)\right) - \theta_j^B\right] = 0 \quad \text{(A.26)}$$

Consider $\alpha = \lambda_j^*$. We know from the proof of Lemma 3 that $E(r) = \overline{R}\left(\lambda_j^*\right)$ is optimal if $\xi_k = 0$, where $E(r) = \overline{R}\left(\lambda_j^*\right)$ implies $\xi_j = 0$. Evaluated at $\alpha = \lambda_j^*$ and $\xi_j = 0$, Eq. (A.25) is

$$\xi_k = \frac{1}{2}\left(F\left(\lambda_j^*, x_k\right) - \frac{x_k^0}{\delta_1}\right)$$

where $x_k = x_k^0 + \delta_1 \xi_k$. Note

$$\frac{\partial F\left(\lambda_j, x_k\right)}{\partial x_k} = g''\left(1 - \theta_k^{s'} + \left(\lambda_j - \theta_j^{s'}\right)\frac{1 - x_k}{x_k}\right)\frac{\left(\lambda_j - \theta_j^{s'}\right)^2}{x_k^3} < 0$$

and therefore

$$\xi_k < \frac{1}{2}\left(F\left(\lambda_j^*, x_k^0\right) - \frac{x_k^0}{\delta_1}\right) < 0$$

where the first inequality follows from $\xi_k > 0$ and the second inequality follows for any $\delta_1 \in \left(0, \frac{x_k^0}{F\left(\lambda_j^*, x_k^0\right)}\right)$. This is a contradiction, hence $\xi_k = 0$ is optimal if $E(r) = \overline{R}\left(\lambda_j^*\right)$. We have now established $\left(E(r), \xi_k\right) = \left(\overline{R}\left(\lambda_j^*\right), 0\right)$ as a solution to k's problem at $\alpha = \lambda_j^*$ so, by concavity, there cannot exist another solution with $\xi_k > 0$ at $\alpha = \lambda_j^*$. Moreover, with $g(\cdot)$ and $h(\cdot)$ well-behaved, Eqs. (A.25) and (A.26) define ξ_j and ξ_k as continuous functions of α so the fact that Eq. (A.25) delivers $\xi_k < 0$ at $\alpha = \lambda_j^*$ implies that it will also deliver $\xi_k < 0$ as α is perturbed above λ_j^*. Therefore, $\xi_k = 0$ for an interval $\alpha \in \left[\lambda_j^*, \lambda_j^* + \varepsilon\right)$, where $\varepsilon > 0$.

The last step is to confirm $\mu_k = 0$. Recall $\lambda_k^* > \lambda_j^*$ from the unregulated equilibrium, so it follows immediately that $\lambda_k^* > \alpha$ when $\alpha = \lambda_j^*$, i.e., bank k is unconstrained at this level

[2] Note that setting $\xi_k = 0$ in Eq. (A.25) and using the result to simplify $\widetilde{G}\left(\xi_j; \alpha\right)$ as defined in Eq. (A.19) delivers the left-hand side of Eq. (A.26) evaluated at $\xi_k = 0$.

of regulation. For $\alpha \in \left[\lambda_j^*, \lambda_j^* + \varepsilon\right)$, Eq. (A.18) becomes

$$\lambda_k = \theta_k^{s'} - \left(\alpha\left(1 - h\left(\xi_j\right)\right) - \theta_j^{s'}\right) \frac{1 - x_k^0 + \delta_1 \xi_j}{x_k^0 - \delta_1 \xi_j}$$

where ξ_j is pinned down by $\widetilde{G}\left(\xi_j; \alpha\right) = 0$, which we recall from the proof of Lemma 3 defines ξ_j as a continuous function of α. Therefore, λ_k is also continuous in α for $\alpha \in \left[\lambda_j^*, \lambda_j^* + \varepsilon\right)$ so it must be the case that k remains unconstrained by regulation (i.e., $\lambda_k > \alpha$ and hence $\mu_k = 0$) as α is perturbed above λ_j^*. ■

Proof of Proposition 3

Given funding share x_k, total credit is

$$TC \equiv \left(1 - \lambda_k\right) x_k + \left(1 - \lambda_j\right)\left(1 - x_k\right)$$

where Eq. (A.18) pins down

$$\left(\lambda_k - \theta_k^{s'}\right) x_k + \left(\lambda_j - \theta_j^{s'}\right)\left(1 - x_k\right) = 0$$

Therefore

$$TC = 1 - \theta_j^{s'} - \left(\theta_k^{s'} - \theta_j^{s'}\right) x_k$$

and

$$\Delta TC = -\left(\theta_k^{s'} - \theta_j^{s'}\right) \Delta x_k$$

where the definition of x_k implies

$$\Delta x_k = \delta_1 \left(\Delta \xi_k - \Delta \xi_j\right)$$

As α is perturbed above λ_j^*, we have $\Delta \xi_j > 0$ and $\Delta \xi_k = 0$ (see Lemmas 3 and 4) and hence $\Delta x_k < 0$. Therefore, $\Delta TC > 0$ if and only if $\theta_k^{s'} > \theta_j^{s'}$, i.e., $s' = B$. Note that $s' = B$ if and only if

$$\theta_k^B x_k^0 + \theta_j^B \left(1 - x_k^0\right) > \theta_k^A x_k^0 + \theta_j^A \left(1 - x_k^0\right)$$

or equivalently

$$x_k^0 > \frac{1}{1 + \frac{\theta_k^B - \theta_k^A}{\theta_j^A - \theta_j^B}} = \widetilde{\pi} \frac{\theta_j^A - \theta_j^B}{\theta_k^B - \theta_j^B}$$

where the equality follows from the assumption that the expected value of the shock is the same for all banks. Intuitively, aggregate feasibility will bind in the state where the price-setting bank borrows if and only if this bank is sufficiently large. ■

Proof of Proposition 4

For the first part of the proposition, we only need to prove the existence of parameters such that the three features hold simultaneously in our model with interbank market power.

Assume the conditions on $h(\cdot)$ in Lemma 3 along with x_k^0 large enough to deliver $s' = B$ as in Proposition 3. This delivers the credit boom.

Next, consider what happens to $E(r)$ as α is varied in the vicinity of λ_j^*, i.e., variations such that $\xi_k = 0$ as per Lemma 4. For $\alpha \geq \lambda_j^*$, the solution is

$$1 + E(r) = g'\left(1 - \alpha\left(1 - h\left(\xi_j\right)\right)\right) - \frac{1}{\alpha h'\left(\xi_j\right)}$$

where ξ_j solves

$$\widetilde{G}\left(\xi_j; \alpha\right) = 0$$

with $\widetilde{G}(\cdot)$ as defined in Eq. (A.19). Differentiate to get

$$\frac{dE(r)}{d\alpha} = -g''\left(1 - \alpha\left(1 - h\left(\xi_j\right)\right)\right)\left(1 - h\left(\xi_j\right) - \alpha h'\left(\xi_j\right)\frac{d\xi_j}{d\alpha}\right) + \frac{1}{\alpha^2 h'\left(\xi_j\right)} + \frac{h''\left(\xi_j\right)}{\alpha\left(h'\left(\xi_j\right)\right)^2}\frac{d\xi_j}{d\alpha}$$

and

$$\frac{d\xi_j}{d\alpha} = -\frac{\widetilde{G}'_\alpha}{\widetilde{G}'_{\xi_j}}$$

Therefore, $\left.\frac{dE(r)}{d\alpha}\right|_{\alpha \to \left(\lambda_j^*\right)^+} > 0$ if and only if

$$\left(g''\left(1 - \lambda_j^*\right) + \frac{h''(0)}{\left(\lambda_j^*\right)^2\left(h'(0)\right)^3}\right)\frac{\widetilde{G}'_\alpha}{\frac{\widetilde{G}'_{\xi_j}}{\lambda_j^* h'(0)}} < \frac{1}{\left(\lambda_j^*\right)^2 h'(0)} - g''\left(1 - \lambda_j^*\right)$$

where the partials $\widetilde{G}'_\alpha$ and $\widetilde{G}'_{\xi_j}$ are evaluated at $\alpha = \lambda_j^*$.

Using the expression for $\widetilde{G}(\cdot)$, we get

$$\begin{aligned}
\widetilde{G}'_\alpha \; = \; & g''\left(1 - \theta_k^{s'} + \left(\lambda_j^* - \theta_j^{s'}\right)\frac{1 - x_k^0}{x_k^0}\right)\frac{1 - x_k^0}{x_k^0} + 2g''\left(1 - \lambda_j^*\right) - g'''\left(1 - \lambda_j^*\right)\left(\lambda_j^* - \theta_j^B\right) \\
& - \frac{1}{\left(\lambda_j^*\right)^2 h'(0)} - \frac{h''(0)}{\left(\lambda_j^*\right)^2\left(h'(0)\right)^3}\left(1 - \frac{2\theta_j^B}{\lambda_j^*}\right) \\
& - \frac{\delta_1}{1 - x_k^0}\left[g''\left(1 - \theta_k^{s'} + \left(\lambda_j^* - \theta_j^{s'}\right)\frac{1 - x_k^0}{x_k^0}\right)\frac{1 - x_k^0}{x_k^0}\frac{\lambda_j^* - \theta_j^{s'}}{x_k^0} - \frac{1}{\lambda_j^* h'(0)}\left(1 - \frac{2\theta_j^B}{\lambda_j^*}\right)\right]\frac{1}{\lambda_j^* h'(0)}
\end{aligned}$$

and

$$\frac{\widetilde{G}'_{\xi_j}}{\lambda_j^* h'(0)} = -g''\left(1-\theta_k^{s'}+\left(\lambda_j^*-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right)\left(1-\frac{\delta_1}{x_k^0\left(1-x_k^0\right)}\frac{\lambda_j^*-\theta_j^{s'}}{\lambda_j^* h'(0)}\right)^2\frac{1-x_k^0}{x_k^0}-2g''\left(1-\lambda_j^*\right)$$
$$+g'''\left(1-\lambda_j^*\right)\left(\lambda_j^*-\theta_j^B\right)-\frac{2h''(0)}{\left(\lambda_j^*\right)^2\left(h'(0)\right)^3}+\frac{\lambda_j^*-\theta_j^B}{\left(\lambda_j^*\right)^3\left(h'(0)\right)^4}\left(h'''(0)-\frac{3\left(h''(0)\right)^2}{h'(0)}\right)$$
$$+\frac{\delta_1}{1-x_k^0}\frac{1+\frac{\delta_1}{1-x_k^0}\frac{\lambda_j^*-\theta_j^B}{\lambda_j^* h'(0)}}{\left(\lambda_j^* h'(0)\right)^2}+\frac{2\delta_1}{1-x_k^0}\frac{\lambda_j^*-\theta_j^B}{\left(\lambda_j^*\right)^3 h'(0)}\frac{h''(0)}{\left(h'(0)\right)^3}$$

at $\alpha=\lambda_j^*$. With $\frac{h''(0)}{(h'(0))^3}\to 0$, the condition for $\left.\frac{dE(r)}{d\alpha}\right|_{\alpha\to\left(\lambda_j^*\right)^+}>0$ reduces to

$$g''\left(1-\lambda_j^*\right)\left(1-\frac{\begin{array}{c}g'''\left(1-\lambda_j^*\right)\left(\lambda_j^*-\theta_j^B\right)-2g''\left(1-\lambda_j^*\right)\\ -g''\left(1-\theta_k^{s'}+\left(\lambda_j^*-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right)\left(1-\frac{\delta_1}{x_k^0\left(1-x_k^0\right)}\frac{\lambda_j^*-\theta_j^{s'}}{\lambda_j^* h'(0)}\right)\frac{1-x_k^0}{x_k^0}\\ +\frac{1}{\left(\lambda_j^*\right)^2 h'(0)}-\frac{\delta_1}{1-x_k^0}\frac{1}{\left(\lambda_j^* h'(0)\right)^2}\left(1-\frac{2\theta_j^B}{\lambda_j^*}\right)\end{array}}{\begin{array}{c}g'''\left(1-\lambda_j^*\right)\left(\lambda_j^*-\theta_j^B\right)-2g''\left(1-\lambda_j^*\right)\\ -g''\left(1-\theta_k^{s'}+\left(\lambda_j^*-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right)\left(1-\frac{\delta_1}{x_k^0\left(1-x_k^0\right)}\frac{\lambda_j^*-\theta_j^{s'}}{\lambda_j^* h'(0)}\right)^2\frac{1-x_k^0}{x_k^0}\\ +\frac{\delta_1}{1-x_k^0}\frac{1+\frac{\delta_1}{1-x_k^0}\frac{\lambda_j^*-\theta_j^B}{\lambda_j^* h'(0)}}{\left(\lambda_j^* h'(0)\right)^2}+\frac{\lambda_j^*-\theta_j^B}{\left(\lambda_j^*\right)^3}\left(\frac{h'''(0)}{(h'(0))^4}-\frac{3(h''(0))^2}{(h'(0))^5}\right)\end{array}}\right)<\frac{1}{\left(\lambda_j^*\right)^2 h'(0)} \tag{A.27}$$

Then, as $h'(0)\to\infty$, it will suffice to have $\left|\frac{h'''(0)}{(h'(0))^4}-\frac{3(h''(0))^2}{(h'(0))^5}\right|\to\infty$.[3] Thus, there exist parameterizations such that $E(r)$ increases as α is perturbed above λ_j^*. Recalling the equilibrium properties of ξ_j around λ_j^* establishes $corr\left(\xi_j, E(r)\right)>0$ as a result of such perturbation.

To establish convergence of on-balance-sheet liquidity ratios, recall $\lambda_j=\alpha\left(1-h\left(\xi_j\right)\right)$ for $\alpha\geq\lambda_j^*$, where $h\left(\xi_j\right)$ is the fraction of funding moved off-balance-sheet via shadow banking. The on-balance-sheet liquidity ratio of j is thus α, which increases as a result of the perturbation. Turn next to the on-balance-sheet ratio of k, which is just λ_k in the vicinity of $\alpha=\lambda_j^*$. From aggregate feasibility,

$$\lambda_k=\theta_k^B-\left(\alpha\left(1-h\left(\xi_j\right)\right)-\theta_j^B\right)\left(\frac{1}{x_k^0+\delta_1\left(\xi_k-\xi_j\right)}-1\right)$$

where we have subbed in the relevant expressions for λ_j and x_k and used $s'=B$. Differen-

[3] An example that satisfies the conditions is $h(\xi)\propto\xi^\gamma$ with $\gamma\in\left(\frac{1}{3},\frac{1}{2}\right)$. Again, this is sufficient, not necessary.

tiation yields:

$$\frac{d\lambda_k}{d\alpha} = -\left(1 - h\left(\xi_j\right) - \alpha h'\left(\xi_j\right)\frac{d\xi_j}{d\alpha}\right)\left(\frac{1}{x_k^0 + \delta_1\left(\xi_k - \xi_j\right)} - 1\right) + \frac{\delta_1\left(\frac{d\xi_k}{d\alpha} - \frac{d\xi_j}{d\alpha}\right)\left(\alpha\left(1 - h\left(\xi_j\right)\right) - \theta_j^B\right)}{\left(x_k^0 + \delta_1\left(\xi_k - \xi_j\right)\right)^2}$$

Therefore, $\frac{d\lambda_k}{d\alpha}\Big|_{\alpha\to\left(\lambda_j^*\right)^+} < 0$ if and only if

$$-\left(1 - \frac{\delta_1\left(\lambda_j^* - \theta_j^B\right)}{\lambda_j^* h'(0)\, x_k^0\left(1 - x_k^0\right)}\right)\frac{\widetilde{G}'_\alpha}{\frac{\widetilde{G}'_{\xi_j}}{\lambda_j^* h'(0)}} < 1 \tag{A.28}$$

where $\widetilde{G}'_\alpha$ and $\widetilde{G}'_{\xi_j}$ are evaluated at $\alpha = \lambda_j^*$. This is the same condition as for $\frac{dE(r)}{d\alpha}\Big|_{\alpha\to\left(\lambda_j^*\right)^+} > 0$ when $\frac{h''(0)}{\left(\lambda_j^*\right)^2\left(h'(0)\right)^3} \to 0$ and $h'(0) \to \infty$, completing the proof of the first part of the proposition.

Remark A.1 *Suppose instead $h'(0)$ is large but not arbitrarily so. This is the case considered in Appendix E. Then the analysis is conducted in the vicinity of $\overline{\alpha}$ instead of λ_j^*, where $\overline{\alpha} > \lambda_j^*$ is the regulation level around which shadow banking emerges in Appendix E, with $\overline{\alpha} \to \lambda_j^*$ as $h'(0) \to \infty$ and $\frac{h''(0)}{(h'(0))^3} \to 0$. Consider $h(\cdot)$ locally linear at zero. The condition for $\frac{dE(r)}{d\alpha}\Big|_{\alpha\to\overline{\alpha}^+} > 0$, i.e., Eq. (A.27) evaluated at $\overline{\alpha}$ instead of λ_j^*, becomes*

$$g''\left(1 - \theta_k^{s'} + \frac{\left(\overline{\alpha} - \theta_j^{s'}\right)\left(1 - x_k^0\right)}{x_k^0}\right)\left(1 - \frac{\delta_1\left(\overline{\alpha} - \theta_j^{s'}\right)\frac{1}{\overline{\alpha}h'(0)}}{x_k^0\left(1 - x_k^0\right)}\right)\left(1 - \frac{\delta_1\left(\overline{\alpha} - \theta_j^{s'}\right)\frac{1 - \overline{\alpha}^2 h'(0) g''(1-\overline{\alpha})}{\overline{\alpha}h'(0)}}{x_k^0\left(1 - x_k^0\right)}\right)$$
$$< \left[g'''\left(1 - \overline{\alpha}\right)\left(\overline{\alpha} - \theta_j^B\right) - g''\left(1 - \overline{\alpha}\right)\left(1 + \delta_1\frac{\frac{\overline{\alpha} - \theta_j^B}{\overline{\alpha}}\left(2 + \frac{\delta_1}{1 - x_k^0}\frac{1}{h'(0)}\right)}{\left(1 - x_k^0\right)h'(0)}\right) + \frac{\delta_1\left(1 + \frac{\delta_1}{1 - x_k^0}\frac{\overline{\alpha} - \theta_j^B}{\overline{\alpha}h'(0)}\right)}{\left(1 - x_k^0\right)\left(\overline{\alpha}h'(0)\right)^2}\right]\frac{x_k^0}{1 - x_k^0}$$

A sufficient condition is

$$1 - \frac{\delta_1\left(\overline{\alpha} - \theta_j^{s'}\right)}{x_k^0\left(1 - x_k^0\right)}\frac{1 - \overline{\alpha}^2 h'(0)\, g''\left(1 - \overline{\alpha}\right)}{\overline{\alpha}h'(0)} \geq 0$$

or equivalently

$$\delta_1\left(\overline{\alpha} - \theta_j^{s'}\right) \leq \frac{x_k^0\left(1 - x_k^0\right)}{\frac{1}{\overline{\alpha}h'(0)} - \overline{\alpha}g''\left(1 - \overline{\alpha}\right)}$$

This defines a non-empty set $D_1 \subset (0, \infty)$ such that $E(r)$ increases as α is perturbed above $\overline{\alpha}$ for any $\delta_1 \in D_1$.

The condition for $\frac{d\lambda_k}{d\alpha}\Big|_{\alpha\to\overline{\alpha}^+} < 0$, i.e., Eq. (A.28) evaluated at $\overline{\alpha}$ instead of λ_j^*, does not *collapse exactly to the condition for* $\left.\frac{dE(r)}{d\alpha}\right|_{\alpha\to\overline{\alpha}^+} > 0$ *if* $h'(0) < \infty$. *Instead, Eq. (A.28) becomes*

$$\frac{\delta_1\left(\overline{\alpha}-\theta_j^B\right)}{x_k^0\left(1-x_k^0\right)}\left[\frac{1}{\overline{\alpha}h'(0)}-2\overline{\alpha}g''(1-\overline{\alpha})+\overline{\alpha}g'''(1-\overline{\alpha})\left(\overline{\alpha}-\theta_j^B\right)\right]+\delta_1\frac{\frac{\overline{\alpha}-\theta_j^B}{\overline{\alpha}}\left(2-\frac{\frac{\delta_1}{x_k^0}\left(1-\frac{2\theta_j^B}{\overline{\alpha}\left(1-x_k^0\right)}\right)}{h'(0)}\right)}{\left(1-x_k^0\right)h'(0)}>1$$

where we have used $s' = B$ *when evaluating* $\widetilde{G}'_\alpha$ *and* $\widetilde{G}'_{\xi_j}$ *at* $\alpha = \overline{\alpha}$. *With* $h'(0)$ *large enough, a sufficient condition is just*

$$\frac{\delta_1\left(\overline{\alpha}-\theta_j^B\right)}{x_k^0\left(1-x_k^0\right)}\left[\frac{1}{\overline{\alpha}h'(0)}-2\overline{\alpha}g''(1-\overline{\alpha})+\overline{\alpha}g'''(1-\overline{\alpha})\left(\overline{\alpha}-\theta_j^B\right)\right]\geq 1$$

or equivalently

$$\delta_1\left(\overline{\alpha}-\theta_j^B\right)\geq\frac{x_k^0\left(1-x_k^0\right)}{\frac{1}{\overline{\alpha}h'(0)}-2\overline{\alpha}g''(1-\overline{\alpha})+\overline{\alpha}g'''(1-\overline{\alpha})\left(\overline{\alpha}-\theta_j^B\right)}$$

This defines a non-empty set $\widetilde{D}_1 \subset (0,\infty)$ *such that* λ_k *decreases as* α *is perturbed above* $\overline{\alpha}$ *for any* $\delta_1 \in \widetilde{D}_1$. *The intersection* $D_1 \cap \widetilde{D}_1$ *is non-empty, completing the proof of the first part of the proposition for the environment of Appendix E.*

For the second part of the proof, consider an alternative model where all banks are price-takers on the interbank market. The FOCs derived earlier for j will now hold for both $i \in \{j,k\}$. Without regulation, the equilibrium will have $g'_j(1-\lambda_j) = 1+E(r) = g'_k(1-\lambda_k)$, where we write $g'_i(\cdot)$ to allow for differences in the long-term investment technology across banks. If $g'_j(\cdot) = g'_k(\cdot)$, then all banks are ex ante identical, x_k does not change in a symmetric equilibrium, and there is no credit boom. Consider next $g'_j(\cdot) > g'_k(\cdot)$, i.e., bank j is more productive than bank k, so that $\lambda_k > \lambda_j$. Since j and k are now just labels, it does not matter to whom we assign the higher $g'_i(\cdot)$. Then $\underline{R}_j(\alpha) > \underline{R}_k(\alpha)$ and shadow banking exists if and only if $E(r) < \underline{R}_j(\alpha)$. Consider a perturbation in the vicinity of $\underline{R}_j(\alpha)$. Specifically, suppose there exists an $\overline{\alpha}'$ and a small perturbation $\varepsilon > 0$ such that $E(r) = \underline{R}_j(\overline{\alpha}')$ when $\alpha = \overline{\alpha}'$ and $E(r) < \underline{R}_j(\overline{\alpha}'+\varepsilon)$ when $\alpha = \overline{\alpha}'+\varepsilon$. Then $\Delta\xi_j > 0$ and $\Delta\xi_k = 0$, which delivers $\Delta TC > 0$ for $s' = B$. However, in the vicinity of $\overline{\alpha}'$, bank k's on-balance-sheet liquidity ratio is either α or λ_k solving $g'_k(1-\lambda_k) = 1+E(r)$ while bank j's ratio is again α. If $\lambda_k = \alpha$, then the perturbation increases the ratios of both banks, i.e., there is no convergence. If instead λ_k solves $g'_k(1-\lambda_k) = 1+E(r)$, then convergence requires a decrease in $E(r)$, which produces $corr\left(\xi_j, E(r)\right) < 0$. This completes the proof of the second part of the proposition. ■

Proof of Proposition 5

The Lagrangian for the planner's problem is

$$\begin{aligned}\mathcal{L}_p &= g\left(1-\lambda_j\right)\left(1-x_k^0\right)+g\left(1-\lambda_k\right)x_k^0-\varepsilon\kappa\left(1-\lambda_k x_k^0-\lambda_j\left(1-x_k^0\right)\right)\\ &\quad+\sum_{s\in\{A,B\}}\upsilon_s^p\left[\left(\lambda_j-\theta_j^s\right)\left(1-x_k^0\right)+\left(\lambda_k-\theta_k^s\right)x_k^0\right]\end{aligned}$$

where $\upsilon_s^p \geq 0$ is the Lagrange multiplier on the aggregate feasibility constraint in state $s \in \{A, B\}$. The first order conditions are

$$g'\left(1-\lambda_j\right)=g'\left(1-\lambda_k\right)=\upsilon_A^p+\upsilon_B^p+\varepsilon\kappa'\left(1-LIQ\right)$$

where $LIQ \equiv \lambda_k x_k^0 + \lambda_j\left(1-x_k^0\right)$. The planner wants perfect risk-sharing, i.e., $\lambda_i = \lambda^p$ for all $i \in \{j, k\}$.

If $\varepsilon = 0$, then

$$g'\left(1-\lambda^p\right)=\upsilon_A^p+\upsilon_B^p$$

With $g'(\cdot) > 0$, the solution requires $\upsilon_A^p + \upsilon_B^p > 0$, which, with $s' = B$, implies $LIQ = \Theta^B\left(x_k^0\right)$. It then follows from Lemma 1 that aggregate liquidity is efficient, i.e., it is pinned down as $LIQ = \Theta^B\left(x_k^0\right)$ in both the decentralized equilibrium and the planner's solution. However, the distribution of liquidity across banks is inefficient in the decentralized equilibrium, i.e., the equilibrium has $\lambda_k > \lambda^p > \lambda_j$ whereas the planner wants $\lambda_k = \lambda_j = \lambda^p$.

If $\varepsilon > 0$, then

$$g'\left(1-\lambda^p\right)=\upsilon_A^p+\upsilon_B^p+\varepsilon\kappa'\left(1-\lambda^p\right)$$

Define λ^* such that

$$\frac{g'\left(1-\lambda^*\right)}{\kappa'\left(1-\lambda^*\right)}=\varepsilon \tag{A.29}$$

With $s' = B$, the planner will want excess liquidity in both non-crisis states (i.e., $\upsilon_A^p = \upsilon_B^p = 0$) in order to reduce the social cost of default in the crisis state if and only if $\lambda^* > \Theta^B\left(x_k^0\right)$, or equivalently (12). Aggregate liquidity in the decentralized equilibrium, which still solves $LIQ = \Theta^B\left(x_k^0\right)$, is then inefficiently low. ■

Proof of Proposition 6

The optimization problem of bank j without the shadow banking technology is simply

$$\max_{\lambda_j\geq\lambda^*}\left\{g\left(1-\lambda_j\right)+\lambda_j-1+\widetilde{\pi}r_A\left(\lambda_j-\theta_j^A\right)+\left(1-\widetilde{\pi}\right)r_B\left(\lambda_j-\theta_j^B\right)\right\}$$

which has Lagrangian

$$\mathcal{L}_j = g(1-\lambda_j) + \lambda_j - 1 + \widetilde{\pi} r_A \left(\lambda_j - \theta_j^A\right) + (1-\widetilde{\pi}) r_B \left(\lambda_j - \theta_j^B\right) + \mu_j(\lambda_j - \lambda^*)$$

and first order condition

$$\lambda_j : \begin{cases} \lambda_j = \lambda^* & if \quad E(r) < \overline{R}(\lambda^*) \\ g'(1-\lambda_j) = 1 + E(r) & if \quad E(r) \geq \overline{R}(\lambda^*) \end{cases}$$

For bank k, the Lagrangian is now

$$\begin{aligned} \mathcal{L}_k &= [g(1-\lambda_k) + \lambda_k - 1] x_k^0 + \left[\widetilde{\pi} r_A \left(\theta_j^A - \lambda_j\right) + (1-\widetilde{\pi}) r_B \left(\theta_j^B - \lambda_j\right)\right] \left(1 - x_k^0\right) \\ &\quad + \mu_k (\lambda_k - \lambda^*) x_k^0 + \sum_{s \in \{A,B\}} \upsilon_s \left[\left(\lambda_j - \theta_j^s\right)\left(1 - x_k^0\right) + \left(\lambda_k - \theta_k^s\right) x_k^0\right] \end{aligned}$$

and the first order condition for λ_k is

$$0 = \frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \left(-g'(1-\lambda_k) + 1 + \upsilon_{s'} + \mu_k\right) x_k^0 \tag{A.30}$$

If $\upsilon_{s'} > 0$, then

$$\lambda_j \left(1 - x_k^0\right) + \lambda_k x_k^0 = \theta_j^{s'} + \left(\theta_k^{s'} - \theta_j^{s'}\right) x_k^0 \equiv \Theta^{s'}\left(x_k^0\right)$$

But then the regulatory constraints $\lambda_j \geq \lambda^*$ and $\lambda_k \geq \lambda^*$ imply $\Theta^{s'}(x_k^0) \geq \lambda^*$, which is false by Condition (12) and the consideration of $s' = B$. Therefore, $\upsilon_{s'} = 0$.

There are two implications of $\upsilon_{s'} = 0$. First, $\mu_k > 0$ from Eq. (A.30) and the properties of $g'(\cdot)$. This then implies $\lambda_k = \lambda^*$. Second,

$$\frac{\partial \mathcal{L}_k}{\partial r_s} \propto \theta_j^s - \lambda_j - E(r) \frac{\partial \lambda_j}{\partial E(r)}$$

so $\frac{\partial \mathcal{L}_k}{\partial r_B} < 0$ follows from $\lambda_j \geq \lambda^* > \theta_j^B$ and $\frac{\partial \lambda_j}{\partial E(r)} \geq 0$. Therefore, $r_B = 0$ and $E(r) = \widetilde{\pi} r_A \leq \widetilde{\pi} \overline{r}_A$.

If $\widetilde{\pi}\overline{r}_A \leq \overline{R}(\lambda^*)$, then $\lambda_j = \lambda^*$ and the proof is complete. If instead $\widetilde{\pi}\overline{r}_A > \overline{R}(\lambda^*)$, then any $E(r) > \overline{R}(\lambda^*)$ would imply $g'(1-\lambda_j) = 1 + E(r)$ and

$$\frac{\partial \mathcal{L}_k}{\partial r_A} \overset{sign}{=} 1 - g'(1-\lambda_j) - g''(1-\lambda_j)\left(\theta_j^A - \lambda_j\right)$$

with

$$\lim_{E(r) \to \overline{R}(\lambda^*)^+} \frac{\partial \mathcal{L}_k}{\partial r_A} \overset{sign}{=} 1 - g'(1-\lambda^*) - g''(1-\lambda^*)\left(\theta_j^A - \lambda^*\right)$$

so $g''(1-\lambda^*) \geq -\frac{g'(1-\lambda^*)-1}{\theta_j^A - \lambda^*}$ (with $g'''(\cdot)$ small so that the problem is concave) would be enough to rule out $E(r) > \overline{R}(\lambda^*)$. Thus, $E(r) \leq \overline{R}(\lambda^*)$ and $\lambda_j = \lambda^*$. ■

Proof of Proposition 7

Without interbank market power, all banks are ex ante identical, so $\lambda_i = \lambda$ and $\xi_i = \xi$ for $i \in \{j,k\}$ and aggregate feasibility requires $\lambda \geq \max\left\{\Theta^A\left(x_k^0\right), \Theta^B\left(x_k^0\right)\right\}$. Any competitive equilibrium will have $r_A = r_B = 0$ if $\lambda > \max\left\{\Theta^A\left(x_k^0\right), \Theta^B\left(x_k^0\right)\right\}$. Eqs. (8) and (9) then imply

$$g'(1-\lambda) = 1 + \frac{1-\rho}{\alpha h'(\xi)}$$

which, with $h'(0) \to \infty$ and $g'(\cdot) > 1$, requires $\xi > 0$. This further implies $\rho = 0$ and $\lambda = \alpha(1-h(\xi))$ so α^* solving

$$\alpha^* h'\left(h^{-1}\left(1 - \frac{\lambda^*}{\alpha^*}\right)\right) = \frac{1}{g'(1-\lambda^*) - 1}$$

implements $\lambda = \lambda^*$ in the absence of interbank market power.

For the rest of the proof, i.e., with interbank market power, recall $\mu_j = \frac{1-\rho_j}{\alpha h'\left(\xi_j\right)}$ from Eq. (9) and $\mu_k = \frac{1-\rho_k-\frac{\delta_1 Z}{x_k}}{\alpha h'(\xi_k)}$ from Eq. (A.15). Consider δ_1 small enough that $\mu_k \approx \frac{1-\rho_k}{\alpha h'(\xi_k)}$, i.e., k's choice of ξ_k is not driven by a strategic desire to get bigger. This allows us to interpret $\xi_i > 0$ as a purely shadow banking action by bank i, i.e., a movement of activity off-balance-sheet to circumvent regulation.

If $\mu_k = 0$, then $\rho_k > 0$ and thus $\xi_k = 0$. Moreover, the first order condition for λ_k is given by Eq. (A.1), which, with $g'(\cdot) > 1$, implies $\upsilon_A + \upsilon_B > 0$. In words, as long as k is not constrained by α, its choice of λ_k will make aggregate feasibility hold with equality in a non-crisis state. With $s' = B$, this implies $\upsilon_B > 0$ and thus $LIQ = \Theta^B(x_k) \leq \Theta^B\left(x_k^0\right)$, where the inequality follows from $\xi_j \geq 0$ and $\xi_k = 0$. Since the planner wants $LIQ > \Theta^B\left(x_k^0\right)$, any α that does not constrain k cannot implement the planner's solution.

If $\mu_j > 0$ and $\mu_k > 0$, then $\lambda_i = \alpha(1-h(\xi_i))$ for $i \in \{j,k\}$. Suppose there exists an α that implements $\lambda_j = \lambda_k = \lambda^*$. Then $\xi_j = \xi_k = \xi^* = h^{-1}\left(1 - \frac{\lambda^*}{\alpha}\right)$ and thus $\mu_j - \mu_k \approx \frac{\rho_k - \rho_j}{\alpha h'(\xi^*)}$. Moreover, from Eqs. (8) and (A.30), $\upsilon_B = E(r) + \mu_j - \mu_k$ and thus $\upsilon_B \approx E(r) + \frac{\rho_k - \rho_j}{\alpha h'(\xi^*)}$. If $\xi^* > 0$, then $\rho_j = \rho_k = 0$. Otherwise, $h'(\xi^*) \to \infty$. Either way, $\upsilon_B \approx E(r) > 0$ for any $\overline{r}_A > 0$ (and possibly also $\overline{r}_A = 0$). Thus, $LIQ = \Theta^B\left(x_k^0\right)$, which is a contradiction since the planner's solution implements $LIQ > \Theta^B\left(x_k^0\right)$.

If instead $\mu_j = 0$, then $g'(1-\lambda_j) = 1 + E(r)$ from Eq. (8), so $\lambda_j = \lambda^*$ is implemented if and only if $E(r) = g'(1-\lambda^*) - 1 = \overline{R}(\lambda^*)$. If $\mu_k > 0$, then $\xi_k > 0$ under the assumption of $h'(0) \to \infty$. Moreover, $\lambda_k = \alpha(1-h(\xi_k)) < \alpha$ so implementing $\lambda_k = \lambda^*$ requires $\alpha > \lambda^*$. But then $E(r) < \overline{R}(\alpha)$, which we know implies $\mu_j > 0$ from Lemma 2. This is a contradiction, completing the proof that an α that implements the planner's solution may not exist with market power.

Consider now welfare, which is

$$\mathcal{W}^e = g\left(1-\lambda_j\right)\left(1-x_k\right) + g\left(1-\lambda_k\right)x_k - \varepsilon\kappa\left(1-LIQ\right)$$

in the decentralized equilibrium. We know from Proposition 3 that LIQ falls as α is perturbed above λ_j^*. Thus,

$$\lim_{\alpha\to\left(\lambda_j^*\right)^+}\frac{\partial\mathcal{W}^e}{\partial\alpha} < 0$$

if $\kappa'\left(1-\Theta^B\left(x_k^0\right)\right)$ is sufficiently large. In words, aggregate welfare falls because the credit boom that accompanies the perturbation of α above λ_j^* further depresses aggregate liquidity. This discussion has implicitly assumed $h'(0)\to\infty$ as in Lemma 3. If instead $h'(0)<\infty$ as in Appendix E, then the relevant threshold for the credit boom is $\overline{\alpha}>\lambda_j^*$. The analysis around $\overline{\alpha}$ is similar to above, i.e.,

$$\lim_{\alpha\to\overline{\alpha}^+}\frac{\partial\mathcal{W}^e}{\partial\alpha} < 0$$

if $\kappa'\left(1-\Theta^B\left(x_k^0\right)\right)$ is sufficiently large. On the interval $\alpha\in\left(\lambda_j^*,\overline{\alpha}\right)$, we recall from Appendix E that $\lambda_j=\alpha$ with $x_k=x_k^0$ and thus

$$\left.\frac{\partial\mathcal{W}^e}{\partial\alpha}\right|_{\alpha\in\left(\lambda_j^*,\overline{\alpha}\right)} = \left[g'\left(1-\theta_k^B+\left(\alpha-\theta_j^B\right)\frac{1-x_k^0}{x_k^0}\right)-g'\left(1-\alpha\right)\right]\left(1-x_k^0\right)>0$$

where we have used the binding aggregate feasibility condition in state B to sub out λ_k from the expression for $\mathcal{W}^e$. In words, a liquidity floor $\alpha\in\left(\lambda_j^*,\overline{\alpha}\right)$ can improve welfare relative to the unregulated equilibrium (i.e., $\mathcal{W}^e$ evaluated at $\alpha=\lambda_j^*$) by creating a more even distribution of liquidity across banks. Of course, the redistribution does not raise aggregate liquidity so welfare is still necessarily lower than in the planner's solution. Moreover, if $\kappa'\left(1-\Theta^B\left(x_k^0\right)\right)$ is large enough, perturbing α above $\overline{\alpha}$ will still push welfare below the unregulated equilibrium.

The proof so far has established the difficulty of using a simple liquidity floor α to achieve the planner's solution in a model with both shadow banking and interbank market power. The planner may then consider bank-specific floors α_i. We sketch out this policy here, which provides a basis for a separate paper on optimal policy and its implementation in a quantitative model.

To fix ideas, consider bank-specific liquidity floors α_j and α_k such that both j and k are constrained, i.e., $\mu_j>0$ and $\mu_k>0$, with $\xi_j>0$ and $\xi_k>0$. Then, by complementary slackness,

$$\lambda_i = \alpha_i\left(1-h\left(\xi_i\right)\right)$$

for $i \in \{j,k\}$ with

$$g'\left(1-\lambda_j\right)-\frac{1}{\alpha_j h'\left(\xi_j\right)}=1+E\left(r\right) \tag{A.31}$$

from Eq. (A.13). Setting $\frac{\partial \mathcal{L}_k}{\partial \lambda_k}=\frac{\partial \mathcal{L}_k}{\partial \xi_k}=\frac{\partial \mathcal{L}_k}{\partial E(r)}=0$ in Eqs. (A.14), (A.15), and (A.16) then gives

$$g'\left(1-\lambda_k\right)=1+\upsilon_B+\mu_k \tag{A.32}$$

$$\mu_k=\frac{1}{\alpha_k h'\left(\xi_k\right)}\left(1-\frac{\delta_1 Z}{x_k}\right) \tag{A.33}$$

$$\left[\alpha_j h'\left(\xi_j\right)\left(\upsilon_B-E\left(r\right)\right)+\frac{\delta_1 Z}{1-x_k}\right]\frac{1}{\alpha_j h'\left(\xi_j\right)g''\left(1-\lambda_j\right)+\frac{h''\left(\xi_j\right)}{\alpha_j\left(h'\left(\xi_j\right)\right)^2}}=\theta_j^B-\lambda_j \tag{A.34}$$

as first order conditions to k's problem, where Z is as defined in the proof of Lemma 3 and we consider $s'=B$ as above.

The planner seeks values of α_j and α_k that implement $\lambda_j=\lambda_k=\lambda^*$. If such values exist, then Eq. (A.31) is

$$E\left(r\right)=g'\left(1-\lambda^*\right)-\frac{1}{\lambda^*}\frac{1-h\left(\xi_j\right)}{h'\left(\xi_j\right)}-1$$

and Eqs. (A.32) and (A.33) combine to give

$$g'\left(1-\lambda^*\right)=1+\upsilon_B+\frac{1}{\lambda^*}\frac{1-h\left(\xi_k\right)}{h'\left(\xi_k\right)}\left(1-\frac{\delta_1 Z}{x_k}\right)$$

where

$$Z\equiv g\left(1-\lambda^*\right)+\lambda^*-1-\xi_k-\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)+E\left(r\right)\left(\lambda^*-\theta_j^B\right)-\frac{\upsilon_B\left(\lambda_j-\theta_j^B\right)}{x_k}$$

We can then isolate

$$\upsilon_B=\frac{\left(1+\frac{x_k}{\delta_1}\frac{\lambda^*}{\lambda^*-\theta_j^B}\frac{h'(\xi_k)}{1-h(\xi_k)}\right)\left(g'\left(1-\lambda^*\right)-1\right)+\frac{g(1-\lambda^*)+\lambda^*-1-\xi_k-\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)-\frac{x_k}{\delta_1}}{\lambda^*-\theta_j^B}-\frac{1}{\lambda^*}\frac{1-h\left(\xi_j\right)}{h'\left(\xi_j\right)}}{\frac{1}{x_k}+\frac{x_k}{\delta_1}\frac{\lambda^*}{\lambda^*-\theta_j^B}\frac{h'(\xi_k)}{1-h(\xi_k)}} \tag{A.35}$$

Finally, Eq. (A.34) is

$$\left(\frac{1}{\lambda^*-\theta_j^B}\frac{1}{1-x_k}\frac{1-h\left(\xi_j\right)}{h'\left(\xi_j\right)}+\lambda^* g''\left(1-\lambda_j\right)+\frac{\frac{h''\left(\xi_j\right)}{1-h\left(\xi_j\right)}}{\lambda^*\left(\frac{h'\left(\xi_j\right)}{1-h\left(\xi_j\right)}\right)^3}\right)\frac{\frac{x_k}{\delta_1}+\frac{\lambda^*-\theta_j^B}{\lambda^* x_k}\frac{1-h(\xi_k)}{h'(\xi_k)}}{\frac{1-h(\xi_k)}{h'(\xi_k)}+\frac{x_k}{1-x_k}\frac{1-h\left(\xi_j\right)}{h'\left(\xi_j\right)}} \tag{A.36}$$

$$=\left(g'\left(1-\lambda^*\right)-1\right)\frac{1-x_k}{x_k}+\frac{1}{\lambda^*}\frac{1-h\left(\xi_j\right)}{h'\left(\xi_j\right)}-\frac{g\left(1-\lambda^*\right)+\lambda^*-1-\xi_k-\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)-\frac{x_k}{\delta_1}}{\lambda^*-\theta_j^B}$$

Recall that x_k is given by Eq. (7), hence Eq. (A.36) pins down a relationship between ξ_k and ξ_j as a function of only parameters.

If $\upsilon_B > 0$, then $\lambda^* = \Theta^B(x_k)$ or equivalently

$$\xi_k = \xi_j + \frac{1}{\delta_1}\left(\frac{\lambda^* - \theta_j^B}{\theta_k^B - \theta_j^B} - x_k^0\right) \tag{A.37}$$

which pins down a second relationship between ξ_k and ξ_j. Otherwise, $\upsilon_B = 0$ and from Eq. (A.35) the second relationship is

$$\begin{aligned} &\left(1 + \frac{x_k}{\delta_1}\frac{\lambda^*}{\lambda^* - \theta_j^B}\frac{h'(\xi_k)}{1 - h(\xi_k)}\right)(g'(1-\lambda^*) - 1) \qquad \text{(A.38)} \\ &= \frac{1}{\lambda^*}\frac{1 - h(\xi_j)}{h'(\xi_j)} - \frac{g(1-\lambda^*) + \lambda^* - 1 - \xi_k - \widetilde{\pi}\overline{r}_A\left(\theta_j^A - \theta_j^B\right) - \frac{x_k}{\delta_1}}{\lambda^* - \theta_j^B} \end{aligned}$$

Optimal policy can take the form of bank-specific liquidity floors for parameterizations of the model where (i) the solution to Eqs. (A.36) and (A.37) has the properties $\xi_j > 0$, $\xi_k > 0$, and $\upsilon_B > 0$ or (ii) the solution to Eqs. (A.36) and (A.38) has the properties $\xi_j > 0$, $\xi_k > 0$, and $\Theta^B(x_k) < \lambda^*$. We leave further analysis of such parameterizations for future study. ■

Proof of Proposition 8

Formally, the Lagrangian for k's problem is now

$$\begin{aligned} \mathcal{L}_k &= [g(1-\lambda_k) + \lambda_k - 1]\, x_k^0 \\ &+ \widetilde{\pi} r_A\left[\left(\theta_j^A - \lambda_j\right)\left(1 - x_k^0\right) - \psi(r_A - r^*)\right] + (1-\widetilde{\pi})\, r_B\left[\left(\theta_j^B - \lambda_j\right)\left(1 - x_k^0\right) - \psi(r_B - r^*)\right] \\ &+ \eta_A(\overline{r}_A - r_A) + \eta_B r_B + \sum_{s\in\{A,B\}} \upsilon_s\left[\left(\lambda_j - \theta_j^s\right)\left(1 - x_k^0\right) + \left(\lambda_k - \theta_k^s\right) x_k^0 + \psi(r_s - r^*)\right] \end{aligned}$$

This is the same as in the proof of Proposition 1, except with the addition of the liquidity injection terms. The counterparts to Eqs. (A.1), (A.2), and (A.3) are:

$$0 = \frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \left(-g'(1-\lambda_k) + 1 + \sum_{s\in\{A,B\}} \upsilon_s\right) x_k^0$$

$$0 = \frac{\partial \mathcal{L}_k}{\partial r_A} = \widetilde{\pi}\left[\theta_j^A - \lambda_j + \left(\sum_{s\in\{A,B\}} \upsilon_s - E(r)\right)\frac{\partial \lambda_j}{\partial E(r)}\right]\left(1 - x_k^0\right) + \psi\left(\upsilon_A + \widetilde{\pi}(r^* - 2r_A)\right) - \eta_A$$

$$0=\frac{\partial \mathcal{L}_k}{\partial r_B}=(1-\widetilde{\pi})\left[\theta_j^B-\lambda_j+\left(\sum_{s\in\{A,B\}}\upsilon_s-E(r)\right)\frac{\partial\lambda_j}{\partial E(r)}\right]\left(1-x_k^0\right)+\psi\left(\upsilon_B+(1-\widetilde{\pi})\left(r^*-2r_B\right)\right)+\eta_B$$

and the counterparts to Eqs. (A.5) and (A.6) are:

$$\eta_A=\frac{\widetilde{\pi}\left(1-x_k^0\right)}{-g''\left(1-\lambda_j\right)}\left[g'\left(1-\lambda_k\right)-g'\left(1-\lambda_j\right)-g''\left(1-\lambda_j\right)\left(\theta_j^A-\lambda_j\right)\right]+\psi\left(\upsilon_A+\widetilde{\pi}\left(r^*-2r_A\right)\right)$$

$$\eta_B=\frac{(1-\widetilde{\pi})\left(1-x_k^0\right)}{g''\left(1-\lambda_j\right)}\left[g'\left(1-\lambda_k\right)-g'\left(1-\lambda_j\right)+g''\left(1-\lambda_j\right)\left(\lambda_j-\theta_j^B\right)\right]-\psi\left(\upsilon_B+(1-\widetilde{\pi})\left(r^*-2r_B\right)\right)$$

If $\eta_A=\eta_B=0$, then

$$\frac{\upsilon_B}{1-\widetilde{\pi}}+r^*-2r_B=\frac{1-x_k^0}{\psi}\left(\frac{g'\left(1-\lambda_k\right)-g'\left(1-\lambda_j\right)}{g''\left(1-\lambda_j\right)}+\lambda_j-\theta_j^B\right)$$

$$\frac{\upsilon_B}{1-\widetilde{\pi}}-\frac{\upsilon_A}{\widetilde{\pi}}+2\left(r_A-r_B\right)=\frac{\left(\theta_j^A-\theta_j^B\right)\left(1-x_k^0\right)}{\psi}$$

Taking limits as $\psi\to\infty$:

$$\frac{\upsilon_B}{1-\widetilde{\pi}}+r^*-2r_B\to 0 \tag{A.39}$$

$$\frac{\upsilon_B}{1-\widetilde{\pi}}-\frac{\upsilon_A}{\widetilde{\pi}}+2\left(r_A-r_B\right)\to 0 \tag{A.40}$$

under the assumption of $\frac{g'(1-\lambda)}{g''(1-\lambda)}$ bounded for $\lambda\in[0,1]$.

Notice that Eq. (13) is violated if $r_s<r^*$ as $\psi\to\infty$. If instead $r_s>r^*$, then Eq. (13) holds with strict inequality as $\psi\to\infty$, implying $\upsilon_s=0$. But then Eq. (A.39) implies $r_B\to\frac{r^*}{2}$, which contradicts $r_B>r^*$. This establishes $r_B\to r^*$, and hence $\frac{\upsilon_B}{1-\widetilde{\pi}}\to r^*$ by Eq. (A.39), as $\psi\to\infty$. Substituting into Eq. (A.40) then implies:

$$2r_A-r^*-\frac{\upsilon_A}{\widetilde{\pi}}\to 0 \tag{A.41}$$

If $r_A>r^*$ as $\psi\to\infty$, then $\upsilon_A=0$, which implies $r_A\to\frac{r^*}{2}$ by Eq. (A.41), contradicting $r_A>r^*$. Therefore, $r_A\to r^*$, and hence $\frac{\upsilon_A}{\widetilde{\pi}}\to r^*$ by Eq. (A.41), as $\psi\to\infty$.

We have now established $E(r)\to r^*$ and $\upsilon_A+\upsilon_B\to r^*$ as $\psi\to\infty$. Hence, Eq. (3) and $\frac{\partial\mathcal{L}_k}{\partial\lambda_k}=0$ deliver $\lambda_j=\lambda_k=\lambda$, where λ solves $g'(1-\lambda)=1+r^*$. The central bank then just needs to set $r^*=g'(1-\lambda^*)-1$ where λ^* solves the planner's problem.

We make two remarks to conclude the proof. First, while aggregate feasibility binds in both non-crisis states (i.e., $\upsilon_s>0$),

$$\lim_{\psi\to\infty}\psi\left(r_s-r^*\right)=\theta_j^s\left(1-x_k^0\right)+\theta_k^s x_k^0-\lambda^*<0$$

to achieve the constrained efficient amount of aggregate liquidity. Second, it remains to verify $\eta_A = \eta_B = 0$. The conjecture of $\eta_B = 0$ is validated by $r^* > 0$. We then just need $r^* \leq \min\{\overline{r}_A, \overline{r}_B\}$, which is trivial under the interpretation that $\overline{r}_s$ is the discount window rate set by the central bank in state s, i.e., the central bank just needs to set the discount window rate $\overline{r}_s$ greater than or equal to the policy rate r^*. ■

Appendix B – No Commitment

We again focus on equilibria with $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$, i.e., k lends in state A and borrows in state B. Without commitment, k will set $r_A = \overline{r}_A$ and $r_B = 0$. Eq. (3) then pins down λ_j^n as

$$g'\left(1-\lambda_j^n\right) = 1 + \widetilde{\pi}\overline{r}_A \tag{B.1}$$

The difference relative to the proof of Proposition 1 is that r_B is no longer endogenous.

To see if there is still an (unregulated) equilibrium with $\lambda_k > \lambda_j$, we use Eq. (A.4) to rewrite $\lambda_k > \lambda_j$ as

$$\frac{\lambda_j - \theta_j^{s'}}{x_k^0} < \theta_k^{s'} - \theta_j^{s'}$$

Define $\overline{\lambda} \equiv \frac{\theta_j^A\theta_k^B - \theta_k^A\theta_j^B}{\theta_j^A - \theta_k^A + \theta_k^B - \theta_j^B}$. If $s' = A$, i.e., $x_k^0 \le \underline{x}_k^0$ with $\underline{x}_k^0 \equiv \frac{\theta_j^A - \theta_j^B}{\theta_j^A - \theta_k^A + \theta_k^B - \theta_j^B}$ as defined in the main text, then $\lambda_k > \lambda_j$ if and only if $x_k^0 < \frac{\theta_j^A - \lambda_j}{\theta_j^A - \theta_k^A}$ or equivalently $x_k^0 < \underline{x}_k^0 + \frac{\overline{\lambda} - \lambda_j}{\theta_j^A - \theta_k^A}$. If $s' = B$, i.e., $x_k^0 \ge \underline{x}_k^0$, then $\lambda_k > \lambda_j$ if and only if $x_k^0 > \frac{\lambda_j - \theta_j^B}{\theta_k^B - \theta_j^B}$ or equivalently $x_k^0 > \underline{x}_k^0 - \frac{\overline{\lambda} - \lambda_j}{\theta_k^B - \theta_j^B}$. Notice that $\lambda_k > \lambda_j$ will be true for any s' if $\lambda_j < \overline{\lambda}$. It is straightforward to show $\overline{\lambda} \in \left(\theta_j^B, \theta_j^A\right)$ so, with $g''(\cdot) < 0$, the following conditions are sufficient for $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ and $\lambda_k > \lambda_j$ in the model without commitment:

$$g'\left(1-\theta_j^B\right) < 1 + \widetilde{\pi}\overline{r}_A$$

$$g'\left(1 - \frac{\theta_j^A\theta_k^B - \theta_k^A\theta_j^B}{\theta_j^A - \theta_k^A + \theta_k^B - \theta_j^B}\right) > 1 + \widetilde{\pi}\overline{r}_A$$

Starting from such an equilibrium, we also obtain the credit boom result in Proposition 3. Assume $h'(0) \to \infty$ as in Lemma 3. $E(r)$ does not respond to α in the absence of commitment, i.e., $E(r)$ always equals $\widetilde{\pi}\overline{r}_A$, so, from the definition of λ_j^n in Eq. (B.1), we can write $E(r) = \overline{R}\left(\lambda_j^n\right)$. A perturbation of α above λ_j^n then immediately implies $\xi_j > 0$ from Lemma 2, where ξ_j solves

$$g'\left(1-\alpha\left(1-h\left(\xi_j\right)\right)\right) - \frac{1}{\alpha h'\left(\xi_j\right)} = 1 + \widetilde{\pi}\overline{r}_A$$

with $\lambda_j = \alpha\left(1-h\left(\xi_j\right)\right)$. To establish $\xi_k = 0$ in the vicinity of $\alpha = \lambda_j^n$, we follow the proof of Lemma 4. In brief, if $\mu_k = 0$, then, at $\alpha = \lambda_j^n$, setting $\frac{\partial \mathcal{L}_k}{\partial \xi_k} = 0$ in Eq. (A.15) delivers $\xi_k < 0$ for any $\delta_1 \in \left(0, \frac{x_k^0}{F\left(\lambda_j^n, x_k^0\right)}\right)$. This contradicts $\frac{\partial \mathcal{L}_k}{\partial \xi_k} = 0$ so $\xi_k = 0$ at $\alpha = \lambda_j^n$ and, by continuity, as α is perturbed above λ_j^n. Continuity together with the starting point $\lambda_k^n > \lambda_j^n$ then confirms $\mu_k = 0$ for this perturbation. The credit boom result then follows exactly as in the proof of Proposition 3 for values of x_k^0 that deliver $s' = B$.

Appendix C – Endogenous $\overline{r}_s$

Consider k's Lagrangian as in the proof of Proposition 1 but with the constraint $r_A \leq \overline{r}_A(\lambda_j)$, where $\overline{r}_A(\lambda_j)$ denotes the solution to $\Upsilon^A(\lambda_j; r_A) = 0$, i.e., $\overline{r}_A(\lambda_j) = \frac{g(1-\lambda_j)+\lambda_j-1}{\theta_j^A-\lambda_j} \geq 0$, which is the interbank rate that would leave bank j with zero profits in state A.

The FOC for λ_k is still given by Eq. (A.1), which again implies Eq. (A.4). The FOCs for r_A and r_B are now

$$0 = \frac{\partial \mathcal{L}_k}{\partial r_A} = \widetilde{\pi}\left[\theta_j^A - \lambda_j + \left(\sum_{s\in\{A,B\}} \upsilon_s - E(r) + \frac{\eta_A \overline{r}'_A(\lambda_j)}{1-x_k^0}\right)\frac{\partial \lambda_j}{\partial E(r)}\right]\left(1-x_k^0\right) - \eta_A$$

$$0 = \frac{\partial \mathcal{L}_k}{\partial r_B} = (1-\widetilde{\pi})\left[\theta_j^B - \lambda_j + \left(\sum_{s\in\{A,B\}} \upsilon_s - E(r) + \frac{\eta_A \overline{r}'_A(\lambda_j)}{1-x_k^0}\right)\frac{\partial \lambda_j}{\partial E(r)}\right]\left(1-x_k^0\right) + \eta_B$$

in place of Eqs. (A.2) and (A.3), which delivers

$$\eta_A = \frac{\widetilde{\pi}\left(1-x_k^0\right)}{-g''(1-\lambda_j)}\left[g'(1-\lambda_k) - g'(1-\lambda_j) - g''(1-\lambda_j)\left(\theta_j^A-\lambda_j\right) + \frac{\eta_A \overline{r}'_A(\lambda_j)}{1-x_k^0}\right]$$

$$\eta_B = -\frac{(1-\widetilde{\pi})\left(1-x_k^0\right)}{-g''(1-\lambda_j)}\left[g'(1-\lambda_k) - g'(1-\lambda_j) + g''(1-\lambda_j)\left(\lambda_j-\theta_j^B\right) + \frac{\eta_A \overline{r}'_A(\lambda_j)}{1-x_k^0}\right]$$

in place of Eqs. (A.5) and (A.6).

If $r_B > 0$, then we obtain

$$g'(1-\lambda_k) - g'(1-\lambda_j) + g''(1-\lambda_j)\left(\lambda_j-\theta_j^B\right) + \frac{\eta_A \overline{r}'_A(\lambda_j)}{1-x_k^0} = 0 \tag{C.1}$$

in place of Eq. (A.7), where $\eta_A > 0$ is still given by Eq. (A.8) and hence $r_A = \overline{r}_A(\lambda_j)$. We can then rewrite Eq. (C.1) as

$$g'\left(1-\theta_k^{s'} + \left(\lambda_j - \theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right) - g'(1-\lambda_j) + g''(1-\lambda_j)\left(\lambda_j-\theta_j^B\right) + \frac{\frac{g(1-\lambda_j)-1+\theta_j^A}{\theta_j^A-\lambda_j} - g'(1-\lambda_j)}{\frac{\theta_j^A-\lambda_j}{\widetilde{\pi}\left(\theta_j^A-\theta_j^B\right)}} = 0 \tag{C.2}$$

This pins down λ_j as a function of parameters. From Eq. (3) and $r_A = \overline{r}_A(\lambda_j)$, we then obtain

$$r_B = \frac{g'(1-\lambda_j) - 1 - \widetilde{\pi}\frac{g(1-\lambda_j)+\lambda_j-1}{\theta_j^A-\lambda_j}}{1-\widetilde{\pi}} \tag{C.3}$$

where λ_j solves Eq. (C.2).

Consider a quadratic specification for the investment return, i.e.,

$$g(y) = g'(0)\,y + \frac{g''(0)}{2}y^2$$

This is the functional form used in our quantitative analysis. Assume $g'(0) > 1$ and $g''(0) < 0$ with $g'(0) + g''(0) > 1$ so that all the properties in Assumption 1 hold. For brevity, also consider $s' = B$ which is equivalent to $x_k^0 \geq \underline{x}_k^0$ from Section 2.3.

Eq. (C.2) simplifies to

$$Q\left(\lambda_j, \theta_j^A\right) \equiv \frac{\left(\lambda_j - \theta_k^B + \frac{\lambda_j - \theta_j^B}{x_k^0}\right)\left(\theta_j^A - \lambda_j\right)^2}{\widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)} + \frac{\left[g'(0) - 1\right]\left(1 - \theta_j^A\right)}{g''(0)} + \frac{\left(1 - \lambda_j\right)^2}{2} - \left(1 - \lambda_j\right)\left(\theta_j^A - \lambda_j\right) = 0$$

where

$$\frac{\partial Q}{\partial \lambda_j} = \left(1 + \frac{1}{x_k^0}\right)\frac{\left(\theta_j^A - \lambda_j\right)^2}{\widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)} - 2\left(\lambda_j - \theta_k^B + \frac{\lambda_j - \theta_j^B}{x_k^0}\right)\frac{\theta_j^A - \lambda_j}{\widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)} + \left(\theta_j^A - \lambda_j\right)$$

$$\frac{\partial Q}{\partial \theta_j^A} = \left(\lambda_j - \theta_k^B + \frac{\lambda_j - \theta_j^B}{x_k^0}\right)\left(2 - \frac{\theta_j^A - \lambda_j}{\theta_j^A - \theta_j^B}\right)\frac{\theta_j^A - \lambda_j}{\widetilde{\pi}\left(\theta_j^A - \theta_j^B\right)} - \frac{g'(0) - 1}{g''(0)} - \left(1 - \lambda_j\right)$$

Notice $Q\left(\lambda_j^1, 1\right) = 0$ has a solution

$$\lambda_j^1 = \frac{\theta_k^B + \frac{\theta_j^B}{x_k^0} + \frac{\widetilde{\pi}\left(1 - \theta_j^B\right)}{2}}{1 + \frac{1}{x_k^0}} \in \left(\theta_j^B, 1\right)$$

with

$$\frac{\partial Q}{\partial \lambda_j}\left(\lambda_j^1, 1\right) = \left(1 + \frac{1}{x_k^0}\right)\frac{\left(1 - \lambda_j^1\right)^2}{\widetilde{\pi}\left(1 - \theta_j^B\right)} \neq 0$$

so the properties of λ_j^1 will extend to λ_j solving $Q\left(\lambda_j, \theta_j^A\right) = 0$ for θ_j^A sufficiently high.

From Eq. (A.4), $\lambda_j^1 < \lambda_k^1$ if and only if $x_k^0 > \frac{\lambda_j^1 - \theta_j^B}{\theta_k^B - \theta_j^B}$. Subbing in for λ_j^1, this reduces to $x_k^0 > \frac{\widetilde{\pi}\left(1 - \theta_j^B\right)}{2\left(\theta_k^B - \theta_j^B\right)}$. Recall the definition of $\underline{x}_k^0$ in Section 2.3. Since the expected value of the liquidity shock is the same for all banks, we can rewrite $\underline{x}_k^0 = \frac{\widetilde{\pi}\left(1 - \theta_j^B\right)}{\theta_k^B - \theta_j^B}$ at $\theta_j^A = 1$. Thus, $\lambda_j^1 < \lambda_k^1$ follows immediately from $x_k^0 \geq \underline{x}_k^0$.

Next, $r_B \leq \overline{r}_B\left(\lambda_k^1\right) \equiv \frac{g\left(1 - \lambda_k^1\right) + \lambda_k^1 - 1}{\theta_k^B - \lambda_k^1}$ if and only if

$$g'\left(1 - \lambda_j^1\right) - 1 \leq \widetilde{\pi}\frac{g\left(1 - \lambda_j^1\right) + \lambda_j^1 - 1}{\theta_j^A - \lambda_j^1} + \left(1 - \widetilde{\pi}\right)\frac{g\left(1 - \lambda_k^1\right) + \lambda_k^1 - 1}{\theta_k^B - \lambda_k^1}$$

where $\overline{r}_B(\cdot)$ is the interbank rate that would leave bank k with zero profits in state B. A sufficient condition for $r_B \leq \overline{r}_B\left(\lambda_k^1\right)$ is

$$g'\left(1-\lambda_j^1\right) \leq \widetilde{\pi}\frac{g\left(1-\lambda_j^1\right)}{1-\lambda_j^1} + (1-\widetilde{\pi})\frac{g\left(1-\lambda_k^1\right)}{1-\lambda_k^1}$$

or equivalently

$$\left(1-\lambda_j^1\right) \geq \frac{\widetilde{\pi}}{2}\left(1-\lambda_j^1\right) + \frac{1-\widetilde{\pi}}{2}\left(1-\lambda_k^1\right)$$

which is true since $\lambda_j^1 < \lambda_k^1$.

Finally, to confirm $r_B > 0$ at $\theta_j^A = 1$, we need

$$g'(0) + g''(0)\left(1-\lambda_j^1\right)\frac{1-\frac{\widetilde{\pi}}{2}}{1-\widetilde{\pi}} > 1$$

which will be true for $g'(0)$ sufficiently high. Eq. (C.3) defines r_B as a continuous function of θ_j^A and λ_j so, for $g'(0)$ and θ_j^A sufficiently high, $r_B > 0$ and the properties derived above hold.

Appendix D – k's Funding Share

Here we sketch a simple optimization problem that generates the funding share x_k in Eq. (7). There is a unit mass of ex ante identical savers. Each saver is endowed with X units of funding but does not have access to the investment technology $g(\cdot)$.

Let X_i denote the funding allocated to bank i by the representative saver. Assume that allocating X_i to bank i entails a transaction cost of $a_i X_i + bX_i^2$, where $a_i \geq 0$ and $b > 0$ are constants.[1] Normalize the base interest rate offered by banks to zero and denote by $\widehat{\xi}_i$ the additional return offered by bank i on its off-balance-sheet products. Let h_i denote the fraction of X_i allocated by the representative saver to bank i's off-balance-sheet products at disutility $\sigma(h_i) X_i$, where $\sigma(0) = 0$, $\sigma'(\cdot) > 0$, and $\sigma''(\cdot) > 0$. The saver's optimization problem is then

$$\max_{\{X_i, h_i\}_{i \in \{j,k\}}} \left\{ \begin{array}{c} \int_{j \in [0,1]} \left(X_j + \widehat{\xi}_j h_j X_j - \sigma(h_j) X_j - a_j X_j - bX_j^2 \right) dj \\ + \left(X_k + \widehat{\xi}_k h_k X_k - \sigma(h_k) X_k - a_k X_k - bX_k^2 \right) \end{array} \right\}$$

subject to the budget constraint

$$\int_{j \in [0,1]} X_j dj + X_k \leq X$$

The FOCs with respect to X_i and h_i are

$$1 + \widehat{\xi}_i h_i - \sigma(h_i) - a_i - 2bX_i = \varphi \tag{D.1}$$

and

$$\sigma'(h_i) = \widehat{\xi}_i \tag{D.2}$$

respectively, where $\varphi \geq 0$ is the Lagrange multiplier on the budget constraint.

Use Eq. (D.1) to get

$$X_j = X_k + \frac{a_k - a_j}{2b} + \frac{\widehat{\xi}_j h_j - \widehat{\xi}_k h_k}{2b} + \frac{\sigma(h_k) - \sigma(h_j)}{2b}$$

for any two banks j and k then substitute into the (binding) budget constraint to isolate

$$X_k = \frac{X + \frac{\overline{a_j} - a_k}{2b}}{2} + \frac{\widehat{\xi}_k h_k - \int_{j \in [0,1]} \widehat{\xi}_j h_j dj}{4b} - \frac{\sigma(h_k) - \int_{j \in [0,1]} \sigma(h_j) dj}{4b}$$

[1] We interpret transactions costs broadly. They have been used in many literatures to parsimoniously model imperfect substitutability between products.

where $\overline{a_j} \equiv \int_{j\in[0,1]} a_j dj$. The funding share $x_k \equiv \frac{X_k}{X}$ is then simply

$$x_k = \frac{1 + \frac{\overline{a_j} - a_k}{2bX}}{2} + \frac{\widehat{\xi}_k h_k - \int_{j\in[0,1]} \widehat{\xi}_j h_j dj}{4bX} - \frac{\sigma(h_k) - \int_{j\in[0,1]} \sigma(h_j) dj}{4bX} \tag{D.3}$$

From the main text, bank i pays $\xi_i x_i$ to move funding $h(\xi_i) x_i$ off of its balance sheet and away from regulation. The implied interest rate on off-balance-sheet products is thus

$$\widehat{\xi}_i = \frac{\xi_i}{h(\xi_i)} \tag{D.4}$$

Savers are atomistic so they take ξ_i and the aggregate fraction $h(\xi_i)$ as given. In a symmetric equilibrium, $h(\xi_i) = h_i$, so, using Eqs. (D.2) and (D.4), we obtain $h(\xi_i)$ as the solution to

$$\sigma'(h(\xi_i))\, h(\xi_i) = \xi_i \tag{D.5}$$

and can rewrite Eq. (D.3) as

$$x_k = \frac{1 + \frac{\overline{a_j} - a_k}{2bX}}{2} + \frac{\xi_k - \overline{\xi_j}}{4bX} - \frac{\frac{\sigma(h(\xi_k))}{\sigma'(h(\xi_k))h(\xi_k)}\xi_k - \int_{j\in[0,1]} \frac{\sigma\left(h\left(\xi_j\right)\right)}{\sigma'\left(h\left(\xi_j\right)\right)h\left(\xi_j\right)}\xi_j dj}{4bX} \tag{D.6}$$

where $\overline{\xi_j} \equiv \int_{j\in[0,1]} \xi_j dj$.[2] Notice that the properties of $\sigma(\cdot)$ imply $h'(\xi_i) > 0$ from Eq. (D.5).

Consider $\sigma(h_i) = h_i^{\nu}$ with $\nu > 1$. Then $\frac{\sigma(h_i)}{\sigma'(h_i)h_i} = \frac{1}{\nu}$ and Eq. (D.6) simplifies to

$$x_k = \frac{1 + \frac{\overline{a_j} - a_k}{2bX}}{2} + \frac{\nu - 1}{4bX\nu}\left(\xi_k - \overline{\xi_j}\right)$$

which is the functional form in Eq. (7).[3]

[2] This appendix has abstracted from the idiosyncratic bank shocks described in the main text. They can be added without affecting the derivations. With probability π_A, fraction θ_k^A of savers are hit by idiosyncratic consumption shocks and have to withdraw all of their funding from all banks at $t = 1$ then $\theta_j^A - \theta_k^A$ of the remaining $1 - \theta_k^A$ savers observe a sunspot and withdraw all of their funding from the banks in type j at $t = 1$. With probability π_B, the fraction hit by idiosyncratic shocks is θ_j^B, with $\theta_k^B - \theta_j^B$ of the remaining $1 - \theta_j^B$ savers observing a sunspot and withdrawing all of their funding from bank k at $t = 1$. The savers involved in the sunspots are chosen at random, hence the representative saver has probability $\pi_A \theta_i^A + \pi_B \theta_i^B$ of withdrawing early from bank i. Suppose $\widehat{\xi}_i$ only accrues at $t = 2$. Then Eqs. (D.1) and (D.2) have $\sum_{s\in\{A,B\}} \pi_s (1 - \theta_i^s) \widehat{\xi}_i^s$ in place of $\widehat{\xi}_i$, but the implied interest rate is $\widehat{\xi}_i^s = \frac{\xi_i x_i}{(1-\theta_i^s) h(\xi_i) x_i}$, i.e., bank i pays $\xi_i x_i$ at $t = 2$ on its remaining off-balance sheet products $(1 - \theta_i^s) h(\xi_i) x_i$, so the system defined by Eqs. (D.1), (D.2), and (D.4) is unaffected.

[3] The solution for $h(\xi_i)$ would simplify to $h(\xi_i) = \left(\frac{\xi_i}{\nu}\right)^{\frac{1}{\nu}}$, which satisfies the properties considered in the main text (up to $h(\infty) \to 1$ which will not bind for reasonable parameters) although we do not impose specifically this functional form for $h(\cdot)$.

Appendix E – Equilibrium when $h'(0) < \infty$

We solve k's problem for $\mu_k = 0$ then verify that the solution satisfies $\lambda_k \geq \alpha(1 - h(\xi_k))$. Recall Eqs. (A.16), (A.17), (A.18), and (7):

$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} \theta_j^B - \lambda_j + (\upsilon_{s'} - E(r)) \frac{\partial \lambda_j}{\partial E(r)} - \frac{\delta_1 Z}{1 - x_k} \frac{\partial \xi_j}{\partial E(r)}$$

$$\upsilon_{s'} = g'(1 - \lambda_k) - 1$$

$$\lambda_k = \theta_k^{s'} - \left(\lambda_j - \theta_j^{s'}\right) \frac{1 - x_k}{x_k}$$

$$x_k = x_k^0 + \delta_1 \left(\xi_k - \xi_j\right)$$

Also recall from Lemma 2 that there are three ranges to consider for $E(r)$:

1. If $E(r) > \overline{R}(\alpha)$, then $\xi_j = 0$ and $1 + E(r) = g'(1 - \lambda_j)$ from the proof of Lemma 2, hence
$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} g'(1 - \lambda_k) - g'(1 - \lambda_j) + g''(1 - \lambda_j)\left(\lambda_j - \theta_j^B\right)$$

2. If $E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right)$, then $\xi_j = 0$ and $\lambda_j = \alpha$ from Lemma 2, hence
$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} \theta_j^B - \lambda_j$$

3. If $E(r) < \underline{R}(\alpha)$, then
$$\lambda_j = \alpha\left(1 - h\left(\xi_j\right)\right)$$
and
$$1 + E(r) = g'\left(1 - \alpha\left(1 - h\left(\xi_j\right)\right)\right) - \frac{1}{\alpha h'\left(\xi_j\right)}$$
from the proof of Lemma 2, hence
$$\begin{aligned}\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} {} & \alpha h'\left(\xi_j\right)\left[g'(1 - \lambda_k) - g'(1 - \lambda_j) + g''(1 - \lambda_j)\left(\lambda_j - \theta_j^B\right)\right] \\ & + 1 + \frac{h''\left(\xi_j\right)}{\alpha\left(h'\left(\xi_j\right)\right)^2}\left(\lambda_j - \theta_j^B\right) + \frac{\delta_1 Z}{1 - x_k}\end{aligned}$$
where
$$\begin{aligned} Z = {} & g(1 - \lambda_k) - g'(1 - \lambda_k)\frac{\lambda_j - \theta_j^{s'}}{x_k} + \left(g'(1 - \lambda_j) - \frac{1}{\alpha h'\left(\xi_j\right)}\right)\left(\lambda_j - \theta_j^B\right) - \xi_k \\ & - \widetilde{\pi}\overline{r}_A\left(\theta_j^A - \theta_j^B\right) - \left(1 - \theta_k^{s'}\right) - \left(\theta_j^{s'} - \theta_j^B\right)\end{aligned}$$

We establish the properties at $\delta_1 = 0$ then extend to an interval $\delta_1 \in \left(0, \widetilde{\delta}_1\right)$ by continuity. At $\delta_1 = 0$:

$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} \begin{cases} \alpha h'(\xi_j) G_1\left(\alpha\left(1-h(\xi_j)\right); x_k^0\right) \\ \qquad +1+\frac{h''(\xi_j)}{\alpha\left(h'(\xi_j)\right)^2}\left(\alpha\left(1-h(\xi_j)\right)-\theta_j^B\right) & if \quad E(r) < \underline{R}(\alpha) \\ \theta_j^B - \lambda_j & if \quad E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right) \\ G_1(\lambda_j; x_k^0) & if \quad E(r) > \overline{R}(\alpha) \end{cases}$$

where $G_1(\cdot)$ is as defined in the proof of Proposition 2.

Lemma E.1 *If $\alpha < \lambda_j^*$, then $E(r) > \overline{R}(\alpha)$ dominates $\overline{R}(\alpha)$. Otherwise, $\overline{R}(\alpha)$ dominates any $E(r) > \overline{R}(\alpha)$.*

Proof. Recall $G_1\left(\lambda_j^*; x_k^0\right) = 0$ and $\frac{\partial G_1\left(\lambda_j; x_k^0\right)}{\partial \lambda_j} < 0$ for $\lambda_j \geq \theta_j^B$ from the proof of Proposition 2. Also recall $\lambda_j = \alpha$ at $E(r) = \overline{R}(\alpha)$ from the proof of Lemma 2. Thus,

$$\lim_{E(r)\to\overline{R}(\alpha)^+} \frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} G_1\left(\alpha; x_k^0\right) > G_1\left(\lambda_j^*; x_k^0\right) = 0$$

for $\alpha \in \left(\theta_j^B, \lambda_j^*\right)$, and

$$\lim_{E(r)\to\overline{R}(\alpha)^+} \frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} G_1\left(\alpha; x_k^0\right) \geq g'\left(1-\theta_k^B - \left(\left(\theta_k^{s'}-\theta_k^B\right)+\left(\theta_j^{s'}-\theta_j^B\right)\frac{1-x_k^0}{x_k^0}\right)\right) - g'(1-\alpha) > 0$$

for $\alpha \leq \theta_j^B$. This establishes the first part of the lemma. To establish the second part, remember from the proof of Lemma 2 that $\lambda_j > \alpha$ for any $E(r) > \overline{R}(\alpha)$. Therefore, $\alpha > \lambda_j^*$ implies

$$\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} G_1\left(\lambda_j; x_k^0\right) < G_1\left(\alpha; x_k^0\right) < G_1\left(\lambda_j^*; x_k^0\right) = 0$$

for all $E(r) > \overline{R}(\alpha)$, completing the proof of the lemma. ■

Lemma E.2 *If $\alpha < \theta_j^B$, then $\overline{R}(\alpha)$ dominates any $E(r) \in \left[\underline{R}(\alpha), \overline{R}(\alpha)\right)$. Otherwise, $\underline{R}(\alpha)$ dominates any $E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right]$.*

Proof. Recall $\lambda_j = \alpha$ for $E(r) \in \left(\underline{R}(\alpha), \overline{R}(\alpha)\right)$ from Lemma 2. The lemma then follows immediately from $\frac{\partial \mathcal{L}_k}{\partial E(r)} \overset{sign}{=} \theta_j^B - \lambda_j$ and the fact that k's objective function is continuous. ■

Lemma E.3 *Define* $\overline{\alpha}$ *such that*

$$\lim_{E(r)\to\underline{R}(\overline{\alpha})^-}\frac{\partial\mathcal{L}_k}{\partial E(r)}=0$$

Then $\overline{\alpha}\geq\lambda_j^*$ *if and only if*

$$1+\frac{h''(0)}{(h'(0))^2}\left(1-\frac{\theta_j^B}{\lambda_j^*}\right)\geq 0 \tag{E.1}$$

Impose Condition (E.1). If $\alpha<\overline{\alpha}$*, then* $\underline{R}(\alpha)$ *dominates any* $E(r)<\underline{R}(\alpha)$*. Otherwise,* $E(r)<\underline{R}(\alpha)$ *dominates* $\underline{R}(\alpha)$*.*

Proof. Recall $\lambda_j=\alpha$ and $\xi_j=0$ at $E(r)=\underline{R}(\alpha)$ from the proof of Lemma 2. Thus,

$$\lim_{E(r)\to\underline{R}(\alpha)^-}\frac{\partial\mathcal{L}_k}{\partial E(r)}\overset{sign}{=}\alpha h'(0)\,G_1\left(\alpha;x_k^0\right)+1+\frac{h''(0)}{(h'(0))^2}\left(1-\frac{\theta_j^B}{\alpha}\right)\equiv A(\alpha)$$

If $1+\frac{h''(0)}{(h'(0))^2}\left(1-\frac{\theta_j^B}{\lambda_j^*}\right)=0$, then $\overline{\alpha}=\lambda_j^*$, where we have used $G_1\left(\lambda_j^*;x_k^0\right)=0$. If instead $1+\frac{h''(0)}{(h'(0))^2}\left(1-\frac{\theta_j^B}{\lambda_j^*}\right)>0$, then $\overline{\alpha}>\lambda_j^*$, where we have used $\frac{\partial G_1\left(\lambda_j;x_k^0\right)}{\partial\lambda_j}<0$ for $\lambda_j\geq\theta_j^B$.

Impose Condition (E.1) so that $\overline{\alpha}\geq\lambda_j^*$. Then,

$$A'(\alpha)=h'(0)\,G_1\left(\alpha;x_k^0\right)+\alpha h'(0)\frac{\partial G_1\left(\alpha;x_k^0\right)}{\partial\alpha}+\frac{h''(0)}{(h'(0))^2}\frac{\theta_j^B}{\alpha^2}<0$$

for $\alpha>\overline{\alpha}$, where we have used $\overline{\alpha}\geq\lambda_j^*$ along with $G_1\left(\lambda_j^*;x_k^0\right)=0$ and $\frac{\partial G_1\left(\lambda_j;x_k^0\right)}{\partial\lambda_j}<0$ for $\lambda_j\geq\theta_j^B$. This establishes

$$\lim_{E(r)\to\underline{R}(\alpha)^-}\frac{\partial\mathcal{L}_k}{\partial E(r)}\overset{sign}{=}A(\alpha)<A(\overline{\alpha})=0$$

for $\alpha>\overline{\alpha}$, which is to say $E(r)<\underline{R}(\alpha)$ dominates $\underline{R}(\alpha)$ for $\alpha>\overline{\alpha}$.

To complete the proof, notice:

1. $G_1(\alpha;x_k^0)>0$ for $\alpha\leq\theta_j^B$ (see the proof of Lemma E.1) and thus $A(\alpha)>0$ for $\alpha\leq\theta_j^B$
2. $A(\alpha)>A(\overline{\alpha})=0$ for $\alpha\in\left(\theta_j^B,\overline{\alpha}\right)$

So, by the concavity of k's problem on the interval $E(r)<\underline{R}(\alpha)$, $\underline{R}(\alpha)$ dominates any $E(r)<\underline{R}(\alpha)$ for $\alpha<\overline{\alpha}$. ■

Taken together, these lemmas imply four regions under Condition (E.1):

1. If $\alpha < \theta_j^B$, then the solution is in the interval $E(r) > \overline{R}(\alpha)$, which will recover the unregulated equilibrium, i.e., $E(r) = R^*$.

2. If $\alpha \in \left(\theta_j^B, \lambda_j^*\right)$, then the solution is either $E(r) = \underline{R}(\alpha)$ or $E(r) = R^*$. Evaluating k's objective function at these two candidates:

$$\begin{aligned}\mathcal{L}_k\left(\underline{R}(\alpha)|\xi_k\right) = g\left(1-\theta_k^{s'}+\left(\alpha-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right)x_k^0 - \left(g'(1-\alpha)-\frac{1}{\alpha h'(0)}\right)\left(\alpha-\theta_j^B\right)\left(1-x_k^0\right)\\ -\xi_k x_k^0+\left[\theta_j^{s'}-\theta_j^B+\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)\right]\left(1-x_k^0\right)-\left(1-\theta_k^{s'}\right)x_k^0\end{aligned}$$

   and

$$\begin{aligned}\mathcal{L}_k\left(R^*|\xi_k\right) = g\left(1-\theta_k^{s'}+\left(\lambda_j^*-\theta_j^{s'}\right)\frac{1-x_k^0}{x_k^0}\right)x_k^0 - g'\left(1-\lambda_j^*\right)\left(\lambda_j^*-\theta_j^B\right)\left(1-x_k^0\right)\\ -\xi_k x_k^0+\left[\theta_j^{s'}-\theta_j^B+\widetilde{\pi}\overline{r}_A\left(\theta_j^A-\theta_j^B\right)\right]\left(1-x_k^0\right)-\left(1-\theta_k^{s'}\right)x_k^0\end{aligned}$$

   Notice

$$\mathcal{L}_k\left(\underline{R}\left(\lambda_j^*\right)|\xi_k\right) = \mathcal{L}_k\left(R^*|\xi_k\right)+\frac{\left(\lambda_j^*-\theta_j^B\right)\left(1-x_k^0\right)}{\lambda_j^* h'(0)} > \mathcal{L}_k\left(R^*|\xi_k\right)$$

   Therefore, the unregulated equilibrium is not necessarily recovered for all $\alpha \in \left(\theta_j^B, \lambda_j^*\right)$. Instead

$$\frac{d\mathcal{L}_k\left(\underline{R}(\alpha)|\xi_k\right)}{d\alpha} = \left[G_1\left(\alpha; x_k^0\right)+\frac{\theta_j^B}{\alpha^2 h'(0)}\right]\left(1-x_k^0\right) > 0$$

   for $\alpha \in \left(\theta_j^B, \lambda_j^*\right)$ and there exists an $\alpha_0 \in \left[\theta_j^B, \lambda_j^*\right)$ such that the solution is $E(r) = R^*$ for $\alpha \in \left(\theta_j^B, \alpha_0\right)$ and $E(r) = \underline{R}(\alpha)$ for $\alpha \in \left(\alpha_0, \lambda_j^*\right)$.[1]

3. If $\alpha \in \left(\lambda_j^*, \overline{\alpha}\right)$, then the solution is $E(r) = \underline{R}(\alpha)$, which delivers $\lambda_j = \alpha$ and $\xi_j = 0$.[2]

4. If $\alpha > \overline{\alpha}$, then the solution is in the interval $E(r) < \underline{R}(\alpha)$, which delivers $\xi_j > 0$.

The next step is to show $\lambda_k(\overline{\alpha}) > \overline{\alpha}$ to confirm that regulation does not bind on k in the vicinity of $\overline{\alpha}$, i.e., k stops increasing $E(r)$ and allows shadow banking to emerge before it is

[1] Notice that there is a discontinuity at $\alpha = \alpha_0$. The price-setting bank lowers $E(r)$ below the unregulated equilibrium R^* and constrains bank j. This deviation is not profitable in the absence of regulation because λ_j would fall by too much. But, with regulation, λ_j cannot fall below α, so there is a range of α below the unregulated λ_j^* where k strategically drops $E(r)$ and constrains j. Importantly, though, there is no shadow banking yet (i.e., $\xi_j = 0$ despite $\mu_j > 0$; we will also verify later $\mu_k = 0$).

[2] As α increases within the interval $\left(\lambda_j^*, \overline{\alpha}\right)$, bank k increases $E(r)$ (i.e., $\underline{R}'(\alpha) > 0$) to keep bank j at $\xi_j = 0$ even though j is constrained by the regulation ($\mu_j > 0$). Keeping $\xi_j = 0$ keeps $\lambda_j = \alpha$, so k is incentivizing j to hold more liquidity as α increases, which allows k to hold less without violating aggregate feasibility.

itself constrained. Recall $\lambda_j = \overline{\alpha}$ at $\alpha = \overline{\alpha}$, so aggregate feasibility in the binding state pins down

$$\lambda_k(\overline{\alpha}) \equiv \theta_k^{s'} - \left(\overline{\alpha} - \theta_j^{s'}\right)\left(\frac{1}{x_k^0} - 1\right)$$

It will suffice to bound the distance between λ_j^* and $\overline{\alpha}$ since, by continuity,

$$\lim_{\overline{\alpha} \to \lambda_j^*} \left[\lambda_k(\overline{\alpha}) - \overline{\alpha}\right] = \lambda_k^* - \lambda_j^* > 0$$

Recall from Lemma E.3 and its proof that $\overline{\alpha}$ solves $A(\overline{\alpha}) = 0$, i.e.,

$$\overline{\alpha} h'(0) G_1\left(\overline{\alpha}; x_k^0\right) + 1 + \frac{h''(0)}{(h'(0))^2}\left(1 - \frac{\theta_j^B}{\overline{\alpha}}\right) = 0$$

Also recall $G_1\left(\lambda_j^*; x_k^0\right) = 0$, hence

$$G_1\left(\lambda_j^*; x_k^0\right) - G_1\left(\overline{\alpha}; x_k^0\right) = \frac{1 + \frac{h''(0)}{(h'(0))^2}\left(1 - \frac{\theta_j^B}{\overline{\alpha}}\right)}{\overline{\alpha} h'(0)}$$

Clearly, $\overline{\alpha} \to \lambda_j^*$ as $h'(0) \to \infty$ (assuming $\frac{h''(0)}{(h'(0))^2}$ finite and/or $\frac{h''(0)}{(h'(0))^3} \to 0$), so $h'(0)$ high enough will ensure that k is not constrained by regulation (i.e., $\mu_k = 0$) at $\overline{\alpha}$.[3] We also recall $\lambda_j = \alpha$ for $\alpha \in (\alpha_0, \overline{\alpha})$, so, by aggregate feasibility, λ_k falls with α over this interval. k not constrained at $\overline{\alpha}$ thus implies k not constrained for any $\alpha < \overline{\alpha}$.

The last step is to establish $\xi_k = 0$ for $\alpha \leq \overline{\alpha}$ and as α is perturbed above $\overline{\alpha}$. We have already established $\mu_k = 0$ for these values of α, so Eq. (A.15) becomes $\frac{\partial \mathcal{L}_k}{\partial \xi_k} = (\rho_k - 1) x_k$. If $\xi_k > 0$, then $\rho_k = 0$ and hence $\frac{\partial \mathcal{L}_k}{\partial \xi_k} < 0$ which contradicts $\xi_k > 0$.

Remark E.1 *The derivation of $E(r)$ here has abstracted from the constraint $r_B \geq 0$. Since $E(r)$ is continuous at $\alpha = \overline{\alpha}$, all the local analysis we do around $\overline{\alpha}$ will satisfy $r_B > 0$ if $\underline{R}(\overline{\alpha}) > \widetilde{\pi}\overline{r}_A$. Once again, bounding the distance between λ_j^* and $\overline{\alpha}$ will ensure that the properties of the unregulated equilibrium (in this case $r_B > 0$) carry over to $\overline{\alpha}$. Note*

$$\underline{R}(\overline{\alpha}) \geq \underline{R}\left(\lambda_j^*\right) = g'\left(1 - \lambda_j^*\right) - 1 - \frac{1}{\lambda_j^* h'(0)} = R^* - \frac{1}{\lambda_j^* h'(0)} > \widetilde{\pi}\overline{r}_A - \frac{1}{\lambda_j^* h'(0)}$$

where the first step follows from $\overline{\alpha} \geq \lambda_j^*$ and $\underline{R}'(\cdot) > 0$, the second from the definition of $\underline{R}(\cdot)$, the third from Eq. (3), and the fourth from the fact that the unregulated equilibrium satisfies $r_B > 0$. Thus, for $h'(0)$ sufficiently large, we will have $\underline{R}(\overline{\alpha}) > \widetilde{\pi}\overline{r}_A$. Next, we check $\alpha < \overline{\alpha}$. The lowest $E(r)$ chosen by k on this interval when not constrained by $r_B > 0$ was

[3] Bounding the distance between λ_j^* and $\overline{\alpha}$ also ensures $\lambda_j \in \left(\theta_j^B, \theta_j^A\right)$ at $\overline{\alpha}$.

$\underline{R}(\alpha_0)$, where α_0 solves $\mathcal{L}_k(\underline{R}(\alpha)|\xi_k) = \mathcal{L}_k(R^*|\xi_k)$. Notice $\alpha_0 \to \lambda_j^*$ and $\underline{R}\left(\lambda_j^*\right) \to R^*$ as $h'(0) \to \infty$, hence $h'(0)$ sufficiently large will also ensure $\underline{R}(\alpha_0) > \widetilde{\pi}\overline{r}_A$.

The results are summarized in the following proposition:

Proposition E.1 *Impose Condition (E.1). For $h'(0)$ sufficiently large and $\delta_1 \in \left(0, \widetilde{\delta}_1\right)$, there exists a unique threshold $\overline{\alpha} \geq \lambda_j^*$ such that (i) $\xi_j = \xi_k = 0$ for any $\alpha \leq \overline{\alpha}$ and (ii) $\xi_j > 0 = \xi_k$ as α is perturbed above $\overline{\alpha}$.*

Propositions 3 and 4 in the main text continue to hold but in the vicinity of $\overline{\alpha}$ instead of λ_j^*. Specifically, the proof of Proposition 3 is unchanged and the remark in the proof of Proposition 4 explicitly considers $h'(0) < \infty$, where, for x_k^0 sufficiently large, we see that the intersection $D_1 \cap \widetilde{D}_1$ in the remark will also intersect $\delta_1 \in \left(0, \widetilde{\delta}_1\right)$ on a non-empty set.

Appendix F – Introducing Capital

Bank i has debt funding x_i^0 (fixed for now) and equity funding e_i. It is standard to assume an extra cost of equity issuance to obtain an interior solution. We denote this cost by $\tau(e_i)$, where $\tau(0) = 0$, $\tau'(\cdot) > 0$, and $\tau''(\cdot) > 0$. Equity-holders get the residual value of the bank at $t = 2$, which is what the bank maximizes in expectation.

Since equity is not subject to withdrawal shocks at $t = 1$, aggregate feasibility in state $s \in \{A, B\}$ requires

$$\lambda_k \left(x_k^0 + e_k\right) + \lambda_j \left(x_j^0 + e_j\right) \geq \theta_k^s x_k^0 + \theta_j^s x_j^0$$

where λ_i is still the reserve-to-asset ratio of bank i, which now differs from the reserve-to-deposit ratio $\lambda_i \left(1 + \frac{e_i}{x_i^0}\right)$.

Unregulated Equilibrium

Given the funding x_j^0 and interbank rates r_A and r_B, the optimization problem of the representative bank j is now

$$\max_{\lambda_j, e_j} \left\{\widetilde{\pi}\Upsilon^A \left(\lambda_j, e_j; r_A\right) + \left(1 - \widetilde{\pi}\right) \Upsilon^B \left(\lambda_j, e_j; r_B\right)\right\}$$

where

$$\Upsilon^s \left(\lambda_j, e_j; r_s\right) \equiv g\left(1 - \lambda_j\right)\left(x_j^0 + e_j\right) + \left(1 + r_s\right)\left(\lambda_j \left(x_j^0 + e_j\right) - \theta_j^s x_j^0\right) - \left(1 - \theta_j^s\right) x_j^0 - \tau\left(e_j\right)$$

is the ex post profit of bank j in state $s \in \{A, B\}$. The FOC for λ_j is the same as before, i.e.,

$$g'\left(1 - \lambda_j\right) = 1 + E\left(r\right) \tag{F.1}$$

and the FOC for e_j is

$$\tau'\left(e_j\right) = g\left(1 - \lambda_j\right) + g'\left(1 - \lambda_j\right)\lambda_j \tag{F.2}$$

where $\frac{de_j}{d\lambda_j} = -g''\left(1 - \lambda_j\right)\frac{\lambda_j}{\tau''(e_j)} > 0$ and $\frac{de_j}{dE(r)} = \frac{\lambda_j}{\tau''(e_j)} > 0$.

Consider next bank k, whose Lagrangian is now

$$\begin{aligned}
\mathcal{L}_k \quad = \quad & \left[g\left(1 - \lambda_k\right) + \lambda_k\right]\left(x_k^0 + e_k\right) - x_k^0 \\
& + \widetilde{\pi} r_A \left[\theta_j^A x_j^0 - \lambda_j \left(x_j^0 + e_j\right)\right] + \left(1 - \widetilde{\pi}\right) r_B \left[\theta_j^B x_j^0 - \lambda_j \left(x_j^0 + e_j\right)\right] - \tau\left(e_k\right) \\
& + \eta_A \left(\overline{r}_A - r_A\right) + \eta_B r_B + \sum_{s \in \{A,B\}} \upsilon_s \left[\lambda_k \left(x_k^0 + e_k\right) + \lambda_j \left(x_j^0 + e_j\right) - \theta_k^s x_k^0 - \theta_j^s x_j^0\right]
\end{aligned}$$

The FOC for λ_k is also the same as before, i.e.,

$$g'(1-\lambda_k) = 1 + \sum_{s\in\{A,B\}} \upsilon_s \tag{F.3}$$

which implies

$$\lambda_k\left(x_k^0 + e_k\right) + \lambda_j\left(x_j^0 + e_j\right) = \theta_k^{s'} x_k^0 + \theta_j^{s'} x_j^0 \tag{F.4}$$

The FOC for e_k is

$$\tau'(e_k) = g(1-\lambda_k) + g'(1-\lambda_k)\lambda_k \tag{F.5}$$

and the FOCs for the interbank rates are

$$\frac{\partial \mathcal{L}_k}{\partial r_A} = \widetilde{\pi}\left[\theta_j^A x_j^0 - \lambda_j\left(x_j^0 + e_j\right)\right] + \widetilde{\pi}\left(\sum_{s\in\{A,B\}} \upsilon_s - E(r)\right)\left[\left(x_j^0 + e_j\right)\frac{d\lambda_j}{dE(r)} + \lambda_j \frac{de_j}{dE(r)}\right] - \eta_A$$

$$\frac{\partial \mathcal{L}_k}{\partial r_B} = (1-\widetilde{\pi})\left[\theta_j^B x_j^0 - \lambda_j\left(x_j^0 + e_j\right)\right] + (1-\widetilde{\pi})\left(\sum_{s\in\{A,B\}} \upsilon_s - E(r)\right)\left[\left(x_j^0 + e_j\right)\frac{d\lambda_j}{dE(r)} + \lambda_j \frac{de_j}{dE(r)}\right] + \eta_B$$

Any equilibrium with $r_B > 0$ will then have

$$r_A = \overline{r}_A \tag{F.6}$$

and

$$g'(1-\lambda_k) - g'(1-\lambda_j) = \frac{\lambda_j\left(x_j^0 + e_j\right) - \theta_j^B x_j^0}{\frac{x_j^0 + e_j}{-g''(1-\lambda_j)} + \frac{\lambda_j^2}{\tau''(e_j)}} \tag{F.7}$$

which implies $\lambda_k > \lambda_j$ if j lends in state B, i.e., if the parameters are such that the equilibrium also has $\lambda_j > \frac{\theta_j^B}{1+\frac{e_j}{x_j^0}}$. Note that $\lambda_k > \lambda_j$ further implies $e_k > e_j$ since $g(1-\lambda) + g'(1-\lambda)\lambda$ is increasing in λ, although the ranking of j and k on the ratio $\frac{e_i}{x_i^0}$ is ambiguous.

Comparison to Planner

Given x_j^0 and x_k^0, the Lagrangian for the planner's problem is now

$$\begin{aligned}\mathcal{L}_p &= g(1-\lambda_j)\left(x_j^0 + e_j\right) + g(1-\lambda_k)\left(x_k^0 + e_k\right) - \tau(e_j) - \tau(e_k) - \varepsilon\kappa\left(x_j^0 + x_k^0 - LIQ\right) \\ &\quad + \sum_{s\in\{A,B\}} \upsilon_s^p\left[\lambda_k\left(x_k^0 + e_k\right) + \lambda_j\left(x_j^0 + e_j\right) - \theta_k^s x_k^0 - \theta_j^s x_j^0\right]\end{aligned}$$

where $LIQ \equiv \lambda_k\left(x_k^0 + e_k\right) + \lambda_j\left(x_j^0 + e_j\right)$. The FOCs for λ_i and e_i are respectively

$$g'(1-\lambda_i) = \varepsilon\kappa'\left(x_j^0 + x_k^0 - LIQ\right) + \sum_{s\in\{A,B\}} \upsilon_s^p$$

$$\tau'(e_i) = g(1-\lambda_i) + g'(1-\lambda_i)\lambda_i$$

Thus, $\lambda_i = \lambda$ and $e_i = e$ for $i \in \{j, k\}$. To reduce notation, normalize $x_j^0 + x_k^0 = 1$ and consider x_k^0 such that $s' = B$. Aggregate liquidity in the unregulated equilibrium will be inefficiently low if $\lambda^* (1 + 2e^*) > \Theta^B (x_k^0)$ where λ^* and e^* solve

$$g'(1 - \lambda^*) = \varepsilon \kappa' (1 - \lambda^* (1 + 2e^*))$$

$$\tau'(e^*) = g(1 - \lambda^*) + g'(1 - \lambda^*) \lambda^*$$

Setting $e^* = 0$ would return the exposition of the planner's problem in the main text.

Comparing to the decentralized equilibrium, the inefficiency still comes from the fact that k finds it suboptimal to leave aggregate feasibility slack in both non-crisis states for the benefit of debt-holders in the crisis state when there is limited liability. Moreover, the decentralized equilibrium does not achieve the efficient level of equity when it does not achieve the efficient liquidity ratios. The corrective action could then be taken on either equity or liquidity.

Regulations

With shadow banking action ξ_i, the economic balance sheet of bank i is:

Assets	Liabilities
Liquid: $\lambda_i (x_i + e_i)$	Debt: x_i
Illiquid: $(1 - \lambda_i)(x_i + e_i)$	Off-B/S Debt: $h(\xi_i) x_i$
Off-B/S Illiquid: $h(\xi_i) x_i$	On-B/S Debt: $(1 - h(\xi_i)) x_i$
On-B/S Illiquid: $(1 - \lambda_i)(x_i + e_i) - h(\xi_i) x_i$	Equity: e_i

As before, the accounting balance sheet excludes the off-balance-sheet assets and liabilities.

The effective liquidity requirement is now

$$\lambda_i (x_i + e_i) \geq \alpha (1 - h(\xi_i)) x_i$$

or equivalently

$$\lambda_i \geq \frac{\alpha (1 - h(\xi_i))}{1 + \frac{e_i}{x_i}} \tag{F.8}$$

Suppose the regulator also imposes a floor $\beta \in (0, 1)$ for the ratio of equity to risk-weighted assets. For simplicity, the risk weight is 0 for liquid assets and 1 for (on-balance-sheet) illiquid assets. The effective capital requirement is then

$$e_i \geq \beta [(1 - \lambda_i)(x_i + e_i) - h(\xi_i) x_i]$$

or equivalently

$$\lambda_i \geq \frac{1 - h\left(\xi_i\right) - \left(\frac{1}{\beta} - 1\right)\frac{e_i}{x_i}}{1 + \frac{e_i}{x_i}} \tag{F.9}$$

In other words, both the liquidity requirement and the risk-weighted capital requirement put floors on the liquidity ratio λ_i.

The liquidity requirement will impose the more stringent floor if and only if

$$\frac{e_i}{x_i} > \frac{1-\alpha}{\frac{1}{\beta} - 1}\left(1 - h\left(\xi_i\right)\right)$$

which reduces to

$$\alpha > 1 - \left(\frac{1}{\beta} - 1\right)\frac{e_i^*}{x_i^0}$$

when evaluated at $\xi_i = 0$, where e_i^* is the equity of bank i in the unregulated equilibrium.

At $\xi_i = 0$, the capital requirement imposed on bank i is $\lambda_i \geq 1 - \frac{\frac{1}{\beta}}{1+\frac{x_i^0}{e_i}}$. We showed above that any unregulated equilibrium where j lends at a positive interbank rate in state B will also have the properties $\lambda_k > \lambda_j$ and $e_k > e_j$. Thus, for $x_k^0 \approx x_j^0$, the capital requirement will not bind on k if it binds infinitesimally on j, suggesting that j will have a stronger motive to operate the shadow technology. All else constant, this will decrease k's debt funding x_k, which also decreases the demand for liquidity in state B, i.e., the state in which k borrows to cover withdrawal shocks. If $x_k^0 > \underline{x}_k^0$, then aggregate feasibility binds in state B, so the reduction in liquidity demanded is met in equilibrium by a reduction in the total amount of liquidity held by banks, i.e., $\Delta LIQ = -\delta_1\left(\theta_k^B - \theta_j^B\right)\left(\Delta\xi_j - \Delta\xi_k\right) < 0$. The same intuition delivers $\Delta LIQ < 0$ in response to a liquidity requirement that binds infinitesimally on j.

Normalizing $x_k + x_j = 1$, total credit in the model with equity is $TC \equiv 1 + e_k + e_j - LIQ$, so what happens to total credit will now also depend on what happens to $e_k + e_j$. To explore what happens to $e_k + e_j$, consider a model with both liquidity and capital requirements. The FOC for e_i will be given by

$$\tau'\left(e_i\right) = g\left(1 - \lambda_i\right) + g'\left(1 - \lambda_i\right)\lambda_i + (1 - \beta)\,\mu_i^e$$

where $\mu_i^e \geq 0$ is the Lagrange multiplier on the capital requirement of bank i. If the liquidity requirement imposes the more stringent floor, then $\mu_i^e = 0$ and by first-order approximation

$$\Delta e_i \approx \frac{-g''\left(1-\lambda_i\right)}{\tau''\left(e_i\right)}\lambda_i \Delta\lambda_i$$

where Δe_i and $\Delta\lambda_i$ denote changes from the unregulated equilibrium. With functional forms

$g(y) = g'(0)\,y + \frac{g''(0)}{2}y^2$ and $\tau(e) = \tau'(0)\,e + \frac{\tau''(0)}{2}e^2$, the approximation is simply

$$\Delta e_i \approx \frac{-g''(0)}{\tau''(0)}\lambda_i \Delta\lambda_i$$

which is to say Δe_i will be of second-order importance for $\tau''(0)$ large. In contrast, $\Delta LIQ < 0$ even as $\tau''(0)$ becomes arbitrarily large. Therefore, $\Delta TC > 0$ for large $\tau''(0)$.[1]

Quantitative Results

We now redo the policy experiment in Section 5.3, i.e., increasing the liquidity floor from $\alpha = 0.145$ to $\alpha = 0.25$, using the extended model developed here. The baseline calibration in Section 5.2 introduces a linear operating cost $\phi_i x_i$ into bank i's objective function as well as external liquidity L into the interbank market. We include these ingredients here too.

Appendix I shows that Chinese banks were not constrained by capital requirements from 2007 to 2014, the sample period for the quantitative analysis in Sections 5.2 and 5.3. The capital adequacy ratio (CAR) of small nationally-operating banks (JSCBs) was just above 0.1 in 2007. The CAR of the Big Four was even higher. We set $\beta = 0.1$ to explore the effects of increasing the liquidity floor when the interbank price-takers in our model are at the boundary of both capital and liquidity regulation.

Recall that the reserve-to-deposit ratio of bank i in the model with capital is $\lambda_i\left(1 + \frac{e_i}{x_i}\right)$. The empirical reserve-to-deposit ratios in 2007 are still 0.38 for the Big Four and 0.14 for the JSCBs (see Section 5.2). The average equity-to-deposit ratio, weighted using deposit shares, is 0.094 for banks in the Big Four and 0.102 for the JSCBs. We therefore target $\frac{e_k}{x_k} = 0.094$ and $\frac{e_j}{x_j} = 0.102$, which implies liquidity ratios of $\lambda_k = \frac{0.38}{1.094} = 0.347$ and $\lambda_j = \frac{0.14}{1.102} = 0.127$. We recalibrate θ_k^B and L to match these liquidity ratios, holding constant the other parameters in the baseline calibration. With $x_k = 0.56$ in 2007 (see again Section 5.2), we can also back out values for e_k and e_j.

To calibrate equity costs, we allow the parameters of the cost function $\tau(\cdot)$ to differ across banks, i.e.,

$$\tau_i(e_i) = \tau_{0i}e_i + \frac{\tau_{1i}}{2}e_i^2$$

The first-order conditions for e_i (see Eqs. (F.2) and (F.5)) then give

$$\tau_{0i} + \tau_{1i}e_i = 1 + z - \frac{\gamma}{2}\left(1 - \lambda_i^2\right) \tag{F.10}$$

where z and γ are the parameters of the $g(\cdot)$ function calibrated in Section 5.2. Denote

[1] One may worry that for $\tau''(0)$ too large, e_i will be so small that the liquidity requirement is no longer the more stringent floor. This does not arise in the calibrated model explored next.

by Δr the difference between the average cost of equity and debt for Chinese banks. The average cost of debt in 2007 is well approximated by the deposit rate, which our model normalizes to zero. The average cost of equity is simply $\frac{\tau_i(e_i)}{e_i}$. Assuming Δr to be the same across banks then implies

$$\tau_{0i} + \frac{\tau_{1i}}{2} e_i = 1 + \Delta r \tag{F.11}$$

The calibrated τ_{0i} and τ_{1i} solve Eqs. (F.10) and (F.11).

We set $\Delta r = 4\%$, i.e., the average cost of equity is 4 percentage points higher than the average cost of debt, which is reasonable for the Chinese economy. As a robustness check, we also try $\Delta r = 2\%$.[2]

Table F.1 reports the results. With $\Delta r = 4\%$, the average interbank rate increases by 30 basis points and the aggregate credit-to-savings ratio increases by around 6 percentage points in response to the tightening of liquidity regulation. The increase in the credit-to-savings ratio is just under 6 percentage points when savings is defined excluding equity outstanding and just over 6 percentage points otherwise. We also see from Panel B of the table that banks remain unconstrained by the capital requirement as the liquidity floor is increased. This is because the tightening of liquidity regulation actually relaxes the capital requirement by incentivizing banks to move less liquid assets off the balance sheet, i.e., $\xi_i > 0$ in Eq. (F.9). Both required and actual equity fall, explaining why the increase in the credit-to-savings ratio is somewhat more pronounced when savings includes equity. Setting instead $\Delta r = 2\%$ implies much more convex equity costs, which reduces the response of e_j and e_k to the tightening of liquidity regulation. However, the increase in the average interbank rate and the credit boom result are very robust.

[2] With $\Delta r = 4\%$, the calibrated parameters are $\tau_{0j} = \tau_{0k} = 1.04$, $\tau_{1j} = 0.03n$ (scaled by the number of banks j to make marginal equity costs comparable between the small and big banks), $\tau_{1k} = 0.18$, $\theta_k^B = 0.66$, and $L = 0.12$. With $\Delta r = 2\%$, the calibrated parameters become $\tau_{0j} = 0.995$, $\tau_{0k} = 0.944$, $\tau_{1j} = 0.922n$, $\tau_{1k} = 0.87$, $\theta_k^B = 0.65$, and $L = 0.11$.

Table F.1

Calibration Results

	(1)	(2)	(3)	(4)
	$\Delta r = 4\%$		$\Delta r = 2\%$	
	$\alpha = 0.145$	$\alpha = 0.25$	$\alpha = 0.145$	$\alpha = 0.25$
	Panel A			
Average Interbank Rate, $E(r)$	0.1%	0.4%	0.1%	0.4%
Price-Setter Loan/Deposit, $(1-\lambda_k)\left(1+\frac{e_k}{x_k}\right)$	0.62	0.70	0.62	0.69
Credit/Savings, $(1-\lambda_j)(1+e_j+e_k)-(\lambda_k-\lambda_j)(x_k+e_k)$	82.3%	88.1%	82.3%	88.1%
Credit/(Savings+Equity), $1-\lambda_j-\frac{(\lambda_k-\lambda_j)(x_k+e_k)}{1+e_j+e_k}$	75.0%	81.3%	75.0%	80.4%
	Panel B			
e_j	0.045	0.040	0.045	0.045
required e_j	0.041	0.023	0.041	0.025
e_k	0.053	0.043	0.053	0.051
required e_k	0.040	0.032	0.040	0.033

Appendix G – Interbank Rate Targeting with Application to U.S.

In this appendix, we sketch out a version of the model where the central bank specifically sets its liquidity injections to target $r_A = r_B = r$ for some $r > 0$. The set-up here can be understood as the limiting case $\psi = \infty$. It requires aggregate feasibility to hold with equality in both states, as slackness in state s would imply $r_s = 0$ in any reasonable competitive equilibrium. A competitive equilibrium is now the relevant benchmark because $\psi = \infty$ removes interbank market power.

The U.S. Federal Reserve has a long history of targeting the interest rate in the Fed Funds market, effectively switching back to FFR targeting in 1982, after using M1 targets from October 1979 to October 1982.[1] The FFR on any given interbank trade is negotiated bilaterally between the borrower and the lender, but the Fed adjusts the total amount of liquidity to target the mean of the FFR distribution at each point in time, i.e., the targeted FFR should prevail in expectation for each state of the world, which would render all banks in our model price-takers. We focus for the moment on the period before 2008, away from the zero lower bound (ZLB).

Consider two interbank price-takers, j and k. For brevity, assume $x_k^0 > \underline{x}_k^0$ so that $s' = B$ in the unregulated equilibrium. In words, aggregate withdrawal pressure is higher in the state where k borrows on the interbank market, i.e., $\Theta^B\left(x_k^0\right) > \Theta^A\left(x_k^0\right)$. Recall $\Theta^s\left(x_k\right) \equiv \theta_k^s x_k + \theta_j^s\left(1 - x_k\right)$ with the evolution of x_k governed by Eq. (7).

Although k is a large bank, it now takes as given prices on the interbank market because the central bank targets these prices. Specifically, the central bank sets $r_s = r$ and injects liquidity $Q^s \in \mathbb{R}$ in state $s \in \{A, B\}$, where

$$\left(\lambda_k - \theta_k^s\right) x_k + \left(\lambda_j - \theta_j^s\right)\left(1 - x_k\right) + Q^s = 0$$

defines the aggregate feasibility condition for state $s \in \{A, B\}$. With

$$LIQ \equiv \lambda_k x_k + \lambda_j\left(1 - x_k\right)$$

we can rewrite

$$Q^s = \Theta^s\left(x_k\right) - LIQ$$

If $Q^s < 0$, then the central bank is extracting liquidity from the interbank market in state s.

With both banks price-takers, the FOCs derived for j in the proof of Lemma 2 will now

[1] See Thornton, D. 2006. "When Did the FOMC Begin Targeting the Federal Funds Rate? What the Verbatim Transcripts Tell Us." *Journal of Money, Credit and Banking*, 38(8), pp. 2039-2071.

hold for both $i \in \{j, k\}$. Specifically, the FOCs for the liquidity ratio λ_i and the shadow banking action ξ_i are

$$g'_i(1-\lambda_i) = 1 + r + \mu_i$$

and

$$\mu_i = \frac{1-\rho_i}{\alpha h'(\xi_i)}$$

with complementary slackness conditions

$$\mu_i[\lambda_i - \alpha(1-h(\xi_i))] = 0,\ \ \mu_i \geq 0,\ \ \lambda_i \geq \alpha(1-h(\xi_i))$$

$$\rho_i \xi_i = 0,\ \ \rho_i \geq 0,\ \ \xi_i \geq 0$$

where we write $g'_i(\cdot)$ to allow for differences in the marginal returns across banks.

If $g'_k(\cdot) = g'_j(\cdot)$, then all banks are ex ante identical so $\lambda_i = \lambda$ and $\xi_i = \xi$ for $i \in \{j, k\}$. Thus, $LIQ = \lambda$, where the unregulated equilibrium is $\widehat{\lambda}$ solving $g'\left(1-\widehat{\lambda}\right) = 1 + r$. In the limiting case of $h'(0) \to \infty$, any regulation $\alpha > \widehat{\lambda}$ will produce

$$\lambda = \alpha(1-h(\xi))$$

with $\xi > 0$ solving

$$g'(1-\alpha(1-h(\xi))) = 1 + r + \frac{1}{\alpha h'(\xi)}$$

Differentiate to get

$$\frac{d\lambda}{d\alpha} = \frac{\frac{1}{h'(\xi)} + \frac{h''(\xi)}{(h'(\xi))^3}[1-h(\xi)]}{\alpha^2 g''(1-\alpha(1-h(\xi))) + \frac{h''(\xi)}{(h'(\xi))^3}}$$

If $h'(0) \to \infty$ and $\frac{h''(0)}{(h'(0))^3} \to 0$, then $\frac{d\lambda}{d\alpha}\Big|_{\alpha \to \widehat{\lambda}^+} = 0$ and

$$\left.\frac{d^2\lambda}{d\alpha^2}\right|_{\alpha \to \widehat{\lambda}^+} = \frac{1}{\widehat{\lambda}^3 g''\left(1-\widehat{\lambda}\right)}\left[\frac{h'''(0)}{(h'(0))^4} - \frac{3(h''(0))^2}{(h'(0))^5}\right]$$

where we recall $g''(\cdot) < 0$, hence there will be a (local) credit reduction if $h(\cdot)$ has the property $\frac{h'''(0)}{(h'(0))^4} < \frac{3(h''(0))^2}{(h'(0))^5}$.[2]

[2] The example mentioned in the proof of Proposition 4, $h(\xi) \propto \xi^\gamma$ with $\gamma \in \left(\frac{1}{3}, \frac{1}{2}\right)$, satisfies this condition. If instead $h'(0) < \infty$ as in Appendix E, then there exists a threshold $\overline{\alpha} > \widehat{\lambda}$ such that (i) $\lambda = \alpha$ for any $\alpha \in \left[\widehat{\lambda}, \overline{\alpha}\right]$ and (ii)

$$\left.\frac{d\lambda}{d\alpha}\right|_{\alpha \to \overline{\alpha}^+} = \frac{\frac{1}{h'(0)} + \frac{h''(0)}{(h'(0))^3}}{\overline{\alpha}^2 g''(1-\overline{\alpha}) + \frac{h''(0)}{(h'(0))^3}}$$

For $h''(0)$ not too negative, e.g., $h(\cdot)$ locally linear at zero, $\frac{d\lambda}{d\alpha}\Big|_{\alpha \to \overline{\alpha}^+} < 0$. Thus, there is a local credit boom

In the U.S., there are various differences between large and small banks. We consider a simple one, namely that large banks have a broader set of productive opportunities and thus a higher marginal cost to holding reserves idly on the balance sheet, i.e., $g'_k(\cdot) > g'_j(\cdot)$.

The unregulated equilibrium has liquidity ratios $\left(\widehat{\lambda}_j, \widehat{\lambda}_k\right)$ solving

$$g'_j\left(1-\widehat{\lambda}_j\right) = 1 + r = g'_k\left(1-\widehat{\lambda}_k\right) \tag{G.1}$$

Thus, $g'_j\left(1-\widehat{\lambda}_j\right) > g'_j\left(1-\widehat{\lambda}_k\right)$ from $g'_k(\cdot) > g'_j(\cdot)$. We then conclude $\widehat{\lambda}_k < \widehat{\lambda}_j$ from $g''_j(\cdot) < 0$. In words, the large bank has a lower liquidity ratio than the small bank and is thus more likely to be constrained by a liquidity floor.

At the liquidity floor $\alpha = \widehat{\lambda}_k$, neither bank is constrained by the regulation so the unregulated equilibrium, $\left(\widehat{\lambda}_j, \widehat{\lambda}_k\right)$ with $\widehat{\xi}_j = \widehat{\xi}_k = 0$, obtains. Consider now a liquidity floor $\alpha \in \left(\widehat{\lambda}_k, \widehat{\lambda}_j\right]$. Bank j remains unconstrained so $\xi_j = 0$ and $\lambda_j = \widehat{\lambda}_j$. In contrast, bank k is constrained, i.e., $\mu_k > 0$ and hence

$$g'_k(1-\lambda_k) = 1 + r + \frac{1}{\alpha h'(\xi_k)} \tag{G.2}$$

$$\lambda_k = \alpha(1 - h(\xi_k)) \tag{G.3}$$

with $\xi_k > 0$. Tightening regulation from $\alpha = \widehat{\lambda}_k$ to $\alpha \in \left(\widehat{\lambda}_k, \widehat{\lambda}_j\right]$ will thus produce $\Delta\xi_k > 0$ and $\Delta\xi_j = 0$. The large bank will engage in shadow banking while the small bank will not.

Remember that $\Delta\xi_k > 0 = \Delta\xi_j$ implies $\Delta x_k > 0$ from Eq. (7). A credit boom can then arise if the central bank injects liquidity in state B, as shown next:

- Consider first $Q^A = 0 < Q^B$ at x_k^0, i.e., the interbank market clears in state A without central bank intervention and the central bank injects liquidity as needed in state B to maintain the target r.[3] Then $LIQ = \Theta^A(x_k)$ for x_k around x_k^0 and a small change $\Delta x_k > 0$ from x_k^0 will imply $\Delta LIQ = -\left(\theta_j^A - \theta_k^A\right)\Delta x_k < 0$, i.e., credit to the real economy rises.

- Now consider $Q^A < 0 = Q^B$, i.e., the interbank market clears in state B without central bank intervention and the central bank extracts liquidity as needed in state A to maintain the target r.[4] Then $LIQ = \Theta^B(x_k)$ and $\Delta x_k > 0$ will imply $\Delta LIQ = \left(\theta_k^B - \theta_j^B\right)\Delta x_k > 0$, i.e., credit to the real economy falls.

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from perturbing α above $\overline{\alpha}$. However, the net effect of moving from $\alpha = \widehat{\lambda}$ (which replicates the unregulated equilibrium) to $\alpha = \overline{\alpha} + \varepsilon$ will be a credit reduction for $\varepsilon > 0$ not too large.

[3] Without this injection, $LIQ < \Theta^B(x_k)$, which would push the equilibrium r_B above r.

[4] Without this extraction, $LIQ > \Theta^A(x_k)$, which would push the equilibrium r_A below r.

- The last possibility is the hybrid one, $Q^A < 0 < Q^B$. In this case, $LIQ \in \left(\Theta^A(x_k), \Theta^B(x_k)\right)$ so we need to use the direct definition $LIQ \equiv \lambda_k x_k + \lambda_j(1 - x_k)$ to evaluate ΔLIQ. Consider specifically an increase from $\alpha = \widehat{\lambda}_k$ to $\alpha = \widehat{\lambda}_j$. Then

$$LIQ_{\alpha=\widehat{\lambda}_k} = \widehat{\lambda}_j - \left(\widehat{\lambda}_j - \widehat{\lambda}_k\right) x_k^0$$

$$LIQ_{\alpha=\widehat{\lambda}_j} = \widehat{\lambda}_j - \left(\widehat{\lambda}_j - \lambda_k\right)\left(x_k^0 + \delta_1 \xi_k\right)$$

and thus

$$\Delta LIQ = \left(\lambda_k - \widehat{\lambda}_k\right) x_k^0 - \left(\widehat{\lambda}_j - \lambda_k\right) \delta_1 \xi_k$$

where λ_k and ξ_k are given by Eqs. (G.2) and (G.3) evaluated at $\alpha = \widehat{\lambda}_j$. To fix ideas, consider the family of functions

$$g_i(y) = g_i'(0)\, y + \frac{g''(0)}{2} y^2$$

where $g_k'(0) > g_j'(0) > r$ and $g''(0) < 0$. Use to rewrite Eq. (G.1) as

$$\widehat{\lambda}_k = \widehat{\lambda}_j - \frac{g_k'(0) - g_j'(0)}{-g''(0)} \tag{G.4}$$

and Eq. (G.2) as

$$\lambda_k - \widehat{\lambda}_k = \frac{1}{-g''(0)\,\widehat{\lambda}_j h'(\xi_k)} \tag{G.5}$$

Then,

$$\Delta LIQ = \frac{x_k^0}{-g''(0)\,\widehat{\lambda}_j h'(\xi_k)} - \left(\widehat{\lambda}_j - \lambda_k\right) \delta_1 \xi_k$$

and, using Eq. (G.3),

$$\Delta LIQ = \frac{x_k^0}{-g''(0)\,\widehat{\lambda}_j h'(\xi_k)} - \widehat{\lambda}_j h(\xi_k)\, \delta_1 \xi_k$$

Therefore, $\Delta LIQ < 0$ if and only if

$$h(\xi_k)\, h'(\xi_k)\, \xi_k > \frac{x_k^0}{-g''(0)\left(\widehat{\lambda}_j\right)^2 \delta_1}$$

For any $h(\cdot)$ such that $h(\xi)\, h'(\xi)\, \xi$ is increasing in ξ, the condition for $\Delta LIQ < 0$ amounts to ξ_k sufficiently high at $\alpha = \widehat{\lambda}_j$. Combining Eqs. (G.3), (G.4), and (G.5), this ξ_k solves

$$\widehat{\lambda}_j h(\xi_k) + \frac{1}{-g''(0)\,\widehat{\lambda}_j h'(\xi_k)} = \frac{g_k'(0) - g_j'(0)}{-g''(0)}$$

where the left-hand side is increasing in ξ_k by the properties of $h(\cdot)$. Therefore, $\Delta LIQ < 0$, i.e., increasing α from $\widehat{\lambda}_k$ to $\widehat{\lambda}_j$ will generate a credit boom, if $g_k'(0) - g_j'(0)$ is sufficiently high.

For simplicity, this appendix has assumed that all banks take as given their equilibrium funding shares. This is the same assumption we made for interbank price-takers in the main text (see Appendix J for robustness checks), and it ensures that operation of the shadow technology is not simply driven by a desire to get bigger, i.e., a competitive motive. Without this assumption here, k could have even more incentive to set $\xi_k > 0$, which would only serve to amplify the credit boom derived for $Q^B > 0$. Our objective in this appendix is illustrative, not quantitative, hence we abstract from any competitive motive to avoid unnecessarily complicating the exposition.

The main conclusion from this exposition is that liquidity regulation α can lead to a credit boom in a model with interbank rate targeting if (i) the big bank has higher marginal returns than the small bank, $g_k'(\cdot) > g_j'(\cdot)$, and (ii) the central bank injects liquidity in the interbank market to maintain its target when the big bank borrows from the small, $Q_B > 0$. As k's funding share increases from its shadow banking activities to skirt the regulation, so too does the total amount of liquidity needed to meet withdrawal requests in state B. If the central bank is not injecting any liquidity in state B, then this demand would have to be met in equilibrium by an increase in the total amount of liquidity held by the banks, leading to a credit reduction rather than a credit boom. In Proposition 3, there were no liquidity injections in state B but liquidity regulation led to a credit boom because k's funding share (and thus the total demand for liquidity in state B) decreased from the shadow banking activities of j.[5]

The analysis in this appendix is for $\psi = \infty$. In Proposition 8, we showed that there is an interbank rate target r^* that implements the planner's solution (without the need for any liquidity regulation) when $\psi \to \infty$. That result was based on $g_k'(\cdot) = g_j'(\cdot)$. Extending Proposition 8 to $g_k'(\cdot) > g_j'(\cdot)$, the planner's first order conditions become $g_i'(1 - \lambda_i^p) = \upsilon_A^p + \upsilon_B^p + \varepsilon\kappa'(1 - LIQ^p)$ for each $i \in \{j, k\}$, where $LIQ^p \equiv \lambda_k^p x_k^0 + \lambda_j^p (1 - x_k^0)$. Thus, the planner's solution can still be achieved with $\psi = \infty$ and a target $r = \upsilon_A^p + \upsilon_B^p + \varepsilon\kappa'(1 - LIQ^p) - 1$.

We conclude by noting that the ZLB raises some interesting considerations. If the Fed Funds market ceases to be the marginal short-term funding market at the ZLB, and if the central bank does not engage in sufficient asset purchases to target spreads in other markets, then large banks will have interbank market power. All else constant, this may lead to large

[5]Note that the conditions $g_k'(\cdot) > g_j'(\cdot)$ and $Q_B > 0$ are in addition to $s' = B$, which was assumed at the outset of this appendix. We recall from Proposition 3 that $s' = B$ was both necessary and sufficient for the credit boom in a model with interbank market power.

banks becoming more liquid than small banks; $g'_k(\cdot) > g'_j(\cdot)$ still pushes towards $\lambda_k < \lambda_j$, but the emergence of interbank market power will push towards $\lambda_k > \lambda_j$ as in Proposition 1. Shadow banking may then be pursued by small banks rather than large banks in response to tighter liquidity regulation, generating a credit boom with the same features as in the main text.

Appendix H – Supplementary Material on China's Interbank Market

The material in this appendix supplements the empirical evidence presented in Section 5.1.

Flow of Funds in June 2013

To better understand the market structure and the relative importance of the Big Four on China's interbank repo market, we analyzed anonymized data on each trade that took place during June 2013. The majority of transactions had either an overnight or a seven-day maturity and there was not much variation in collateral or haircuts. Accordingly, we focused on interest rates and loan amounts.

Figure H.1 graphs the interbank network for the main sample, which excludes June 20 and 21. There was a dramatic spike in interbank interest rates on June 20, which we discuss in more detail in the next subsection. Each node in Figure H.1 represents a group of banks. The flow of funds between the nodes is indicated by the direction of the arrows, with thicker arrows signifying more trade. Eigenvector centrality is one way to put numbers on the approximate importance of each of the nodes. It is based on the idea that a central node is connected to other central nodes. We only need to specify an adjacency matrix A that summarizes the connections between the nodes. The centrality of node i is then the i^{th} element of the eigenvector associated with the largest eigenvalue of A. The first column in Table H.1 reports the results when the connection from node i to node s in the adjacency matrix is based on average daily lending from i to s. The second column reports the results when the connection from i to s is based on average daily borrowing by i from s. It is clear from these two columns that the policy banks and the Big Four are the central lending nodes in the main sample.

The third and fourth columns of Table H.1 repeat the eigenvector centrality analysis with adjacency matrices constructed using data from June 20, as opposed to the main sample. The results show minimal change in the centrality of the policy banks on June 20 relative to the main sample. In contrast, the Big Four became much less central on the lending side and much more central on the borrowing side. Therefore, the lending and borrowing decisions of the Big Four may have a dramatic effect on the tightness of the interbank market, even if the policy banks remain a central lending node.

We can also compare the ability of each node in Figure H.1 to impact interbank conditions by calculating the elasticity of total lending by the interbank market with respect to the money that each of these nodes brings into the market. Consider the N nodes in Figure H.1.

Let ε_i^+ denote the money that node i brings into the interbank market and let ε_i^- denote the money that node i takes out of the interbank market. Also let $y_{i,s}$ denote the money that node i lends to node s on the interbank market. The adding-up constraint for each node i is therefore:

$$\sum_s y_{i,s} + \varepsilon_i^- = \sum_s y_{s,i} + \varepsilon_i^+ \tag{H.1}$$

It will be convenient to rewrite in matrix notation. Define $y_i \equiv \sum_s y_{i,s} + \varepsilon_i^-$ and $m_{i,s} \equiv \frac{y_{i,s}}{y_i}$. Also define an $N \times N$ matrix $M = (m_{i,s})$ and $N \times 1$ vectors $Y = (y_i)$ and $E^+ = (\varepsilon_i^+)$. The system of (H.1) for all i is just:

$$Y = M'Y + E^+$$

which can be rearranged to write:

$$Y = [I - M']^{-1} E^+ \tag{H.2}$$

where I is an $N \times N$ identity matrix. Suppose the matrix M and the vector E^+ are fixed. Then, for each node i, we can use (H.2) to calculate the elasticity of total lending by the interbank market, $\sum_s y_s$, to the money that i brings into the interbank market, ε_i^+.

To proceed, we need the matrix M. The $(i,s)^{th}$ element of M is $m_{i,s} \equiv \frac{y_{i,s}}{y_i}$, where $y_i \equiv \sum_s y_{i,s} + \varepsilon_i^-$. For $y_{i,s}$, we used the average daily lending from node i to node s in June 2013, excluding June 20 and 21. The policy banks and the Big Four are net lenders so we assumed $\varepsilon_i^- = 0$ for each of them then used (H.1) to get their respective ε_i^+'s. For each of the other nodes, we assumed that the money it brings into the interbank market (ε_i^+) as a fraction of what the Big Four brings equals the ratio of its deposits to the Big Four's deposits in 2013. We then used (H.1) to get ε_i^- for each of these other nodes.

The results using the main sample are reported in the last column of Table H.1. An elasticity of 0.29 for the Big Four means that, on an average trading day in the main sample, a 1 percent increase in the amount of money brought into the interbank market by the Big Four leads to a 0.29 percent increase in total lending by this market. This is 3.7 times the elasticity for the JSCBs and 0.5 times the elasticity for the policy banks, which is substantial given the quantity adjustments that the Big Four can make. The scale of these adjustments was apparent on June 20. Policy banks brought 72 percent more money into the interbank market than they did on an average trading day in the main sample. Total lending by the interbank market should have then increased by 41 percent, given the elasticity of 0.57 in Table H.1. However, the Big Four brought 183 percent less money into the interbank market than they did on an average trading day in the main sample and, with an elasticity of 0.29, this leads to a 53 percent decrease in total lending by the interbank market, more than enough to offset the efforts of the policy banks.

The June 20 Event

We then studied in more detail the dramatic spike in interbank interest rates that occurred in China on June 20, 2013. The weighted average interbank repo rate hit an unprecedented 11.6% on this date. For comparison, the average across all other trading days in June 2013 was 6.4%, the average in the prior month (May) was 3.0%, and the average in the following month (July) was 3.6%.

A common narrative in China is that interbank conditions tightened on June 20 because the government wanted to discipline the market, either deliberately or by not responding to some market pressures. An analysis of individual transactions will show whether or not this narrative is correct. Our identification strategy here makes use of the fact that China's three policy banks participate in the interbank repo market. The policy banks are agents of the government so the price and quantity of the liquidity that they provide is easily controlled by the government. In contrast, China's big commercial banks have become much more independent since the market-oriented reforms discussed in Section 5.1. If China's interbank repo market tightened at the hands of the government, there should be at least some evidence of restrictive behavior by policy banks relative to other banks on June 20.

The transaction-level data show that this was not the case. The policy banks provided a lot of liquidity to the interbank market at fairly low interest rates, to the point that they became the largest net lenders on June 20. The Big Four, on the other hand, were extremely restrictive, amassing RMB 50 billion of net borrowing by the end of the trading day.

Figure H.2 illustrates the sharp difference between the Big Four and the policy banks in terms of both quantity and price of liquidity provision on June 20. Notice the sizeable increase in policy bank loans and the more moderate nature of policy bank interest rates. Figure H.2 also reveals that much of the increase in policy bank lending on June 20 was absorbed by the Big Four, a fact also visible from the flow of funds depicted in Figure H.3.

Were big banks borrowing because they really needed liquidity? Two pieces of evidence suggest no. First, the Big Four's ratio of gross lending to gross borrowing was 0.7 on June 20, with 71% of the loans directed towards small banks. If the Big Four were in dire need of liquidity, we would expect to see very little outflow. Second, the repo market activities of big banks on June 20 involved a maturity mismatch. Overnight trades accounted for 96% of big bank borrowing but only 83% of big bank lending to small banks. Roughly 80% of policy bank lending to small banks was also at the overnight maturity. If big banks really needed liquidity on June 20, we would expect the maturity of their lending to be closer to the maturity of their borrowing. Instead, it was closer to the maturity offered by policy banks to borrower groups that policy banks and big banks had in common.

The left panel of Figure H.4 shows that big banks also commanded an abnormally high interest rate spread on June 20. In particular, their weighted average lending rate was 266 basis points above their weighted average borrowing rate. This is high relative to other banks: JSCBs and city banks commanded spreads of 113 and 46 basis points respectively. It is also high relative to other days in the sample: on any other day in June 2013, the spread commanded by big banks was between -40 and 58 basis points. Pricing among big banks was also much more uniform than pricing among small banks, both on June 20 and throughout our sample. To this point, we calculated the coefficient of variation (CV) of overnight lending rates offered by banks in different groups. The CV among big banks was 61% of the CV among JSCBs and 21% of the CV among city banks on June 20. Averaging over all trading days in June 2013 yields similar figures, namely 62% and 29% respectively.[1]

The right panel of Figure H.4 shows that JSCBs paid a lot more for non-policy bank loans on June 20 than they did for policy bank loans.[2] There were no major differences in the haircuts imposed by policy banks versus other lenders. It then stands to reason that JSCBs would have liked a higher share of policy bank lending. Instead, they received 20% of what policy banks lent on June 20, down from an average of 28% over the rest of the month. The situation was similar for city and rural banks: they faced large price differentials between policy and non-policy bank loans yet their share of policy bank lending on June 20 was 22%, well below an average of 47% over the rest of the month.

Taken together, the evidence from the June 2013 data indicates that the Big Four can and do change prices on China's interbank market, even controlling for government policy. China's policy banks provided a sizeable amount of liquidity on June 20 but the Big Four prevented interbank rates from falling.

Further NCD Evidence

The empirical evidence in the main text (Section 5.1) used announcements of interbank NCDs from 2016 to 2018 to identify increases in liquidity demand. Table 1 showed that NCD announcements by the Big Four led to higher interbank repo rates on the next trading day. Table H.2 repeats the analysis for each bank individually. Columns (1) to (4) use NCD announcements by each bank in the Big Four. Columns (5) to (16) use NCD announcements by each of the twelve JSCBs. As in Table 1, we control for two lags of the repo rate as well

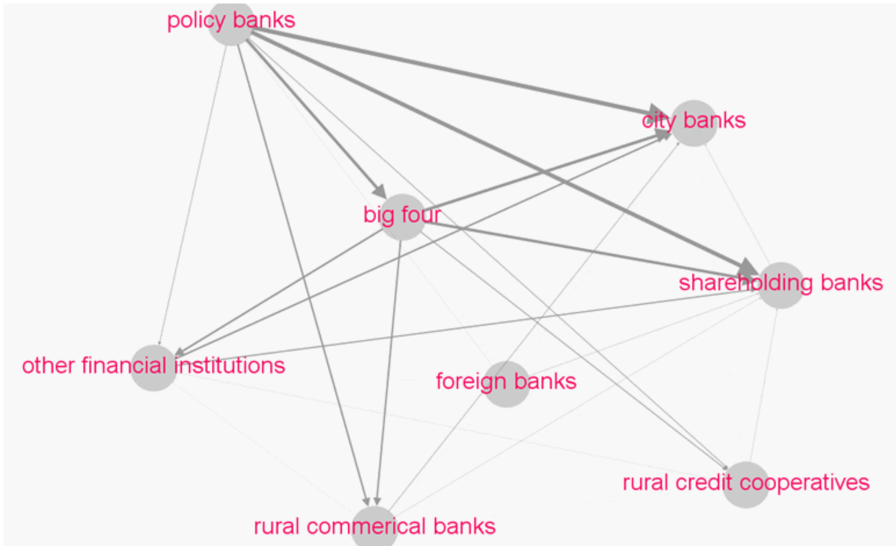
[1] We excluded lending rates charged to policy banks given the proximity of policy banks to the government.

[2] For completeness, the overnight and 7 day maturities shown in the right panel of Figure H.4 were 94% of JSCB borrowing on June 20. They were also 100% of JSCB borrowing from policy banks on this date.

as the required reserve ratio set by the PBOC. The response of the interbank repo rate to NCD announcements by the JSCBs is not statistically different from zero.

Figure H.1

Interbank Network in China, Net Flows



Notes: Based on main sample. Shareholding banks are the JSCBs.

Table H.1

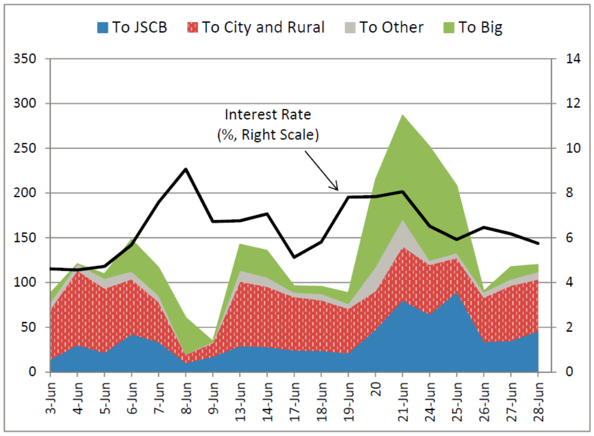
Measures of Bank Importance on Interbank Market

	Eigen-Centrality				Elasticity
	Main Sample		June 20		
	Out	In	Out	In	
Policy Banks	1.00	0.01	1.00	0.07	0.572
Big Four	0.97	0.23	0.56	0.54	0.287
JSCBs	0.67	0.71	0.47	1.00	0.078
City Banks	0.77	1.00	0.33	0.95	0.037
Rural Banks	0.37	0.34	0.20	0.29	0.018
Rural Co-ops	0.18	0.20	0.11	0.12	0.002
Foreign Banks	0.08	0.04	0.02	0.06	0.006
Other	0.97	0.73	0.93	0.73	0.000

Notes: Out is based on lending. In is based on borrowing. Last column is elasticity of total lending by interbank market with respect to money brought into market by node.

Figure H.2: Repo Lending (RMB Billions)

By Policy Banks



By Big Banks

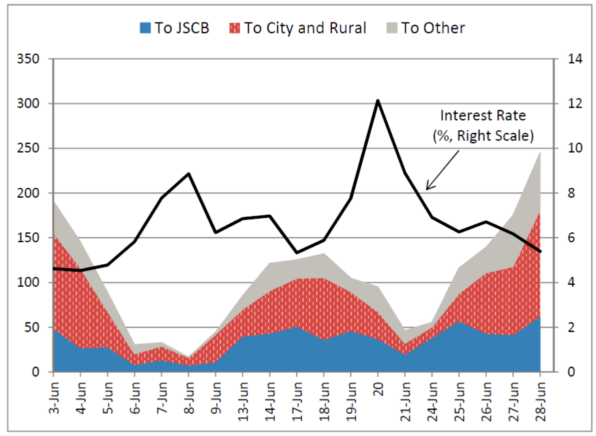


Figure H.3: Interbank Network on June 20, Net Flows

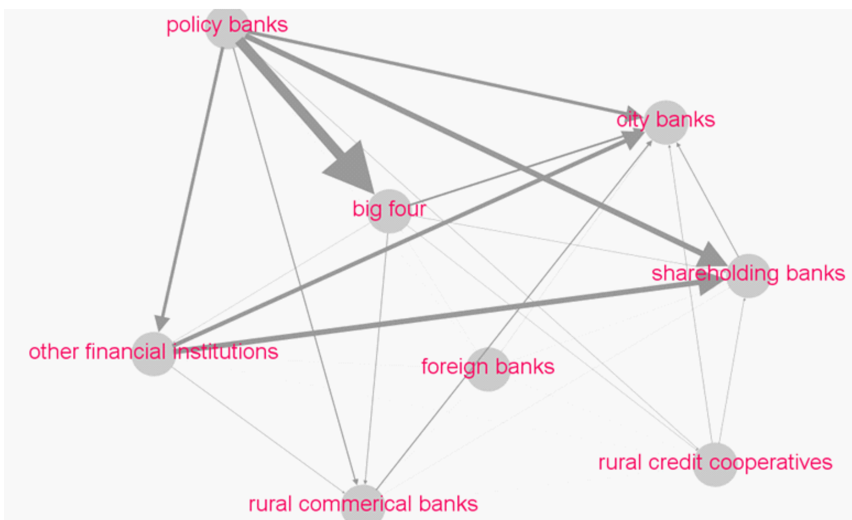
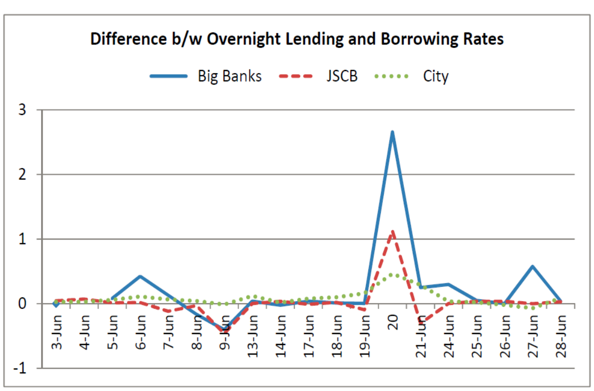


Figure H.4: Interbank Market Spreads



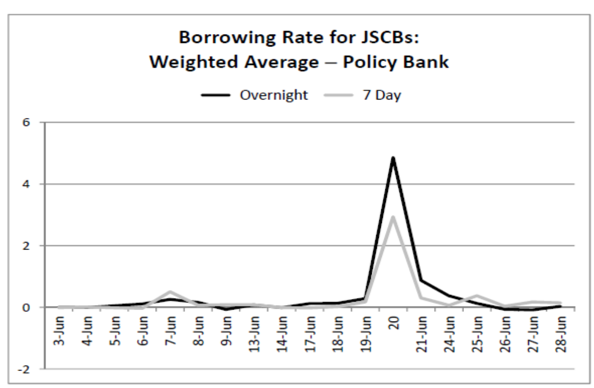


Table H.2

Interbank Repo Rate Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
L1.repo	0.832***	0.834***	0.829***	0.835***	0.835***	0.835***	0.834***	0.835***
	(0.0378)	(0.0380)	(0.0377)	(0.0379)	(0.0379)	(0.0381)	(0.0379)	(0.0379)
L2.repo	0.0769**	0.0893**	0.0711*	0.0889**	0.0892**	0.0876**	0.0852**	0.0884**
	(0.0384)	(0.0382)	(0.0382)	(0.0382)	(0.0382)	(0.0385)	(0.0382)	(0.0381)
RRR	0.203***	0.196***	0.199***	0.197***	0.196***	0.197***	0.191***	0.195***
	(0.0715)	(0.0717)	(0.0711)	(0.0717)	(0.0718)	(0.0717)	(0.0716)	(0.0716)
L.NCD_ABC	0.0258**							
	(0.0116)							
L.NCD_BOC		-0.00583						
		(0.0158)						
L.NCD_CCB			0.0421***					
			(0.0120)					
L.NCD_ICBC				-0.00420				
				(0.0479)				
L.NCD_CMB					-0.00183			
					(0.0108)			
L.NCD_SPDB						0.00381		
						(0.0149)		
L.NCD_CITIC							0.0202	
							(0.0124)	
L.NCD_CIB								-0.0466
								(0.0331)
Observations	748	748	748	748	748	748	748	748
R-squared	0.841	0.840	0.843	0.840	0.840	0.840	0.841	0.841

Table H.2 (continued)

Interbank Repo Rate Regressions

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
L1.repo	0.835***	0.835***	0.835***	0.831***	0.833***	0.835***	0.835***	0.835***
	(0.0379)	(0.0379)	(0.0379)	(0.0381)	(0.0380)	(0.0379)	(0.0379)	(0.0379)
L2.repo	0.0878**	0.0879**	0.0888**	0.0932**	0.0928**	0.0889**	0.0892**	0.0826**
	(0.0384)	(0.0382)	(0.0381)	(0.0385)	(0.0386)	(0.0382)	(0.0382)	(0.0384)
RRR	0.197***	0.198***	0.192***	0.198***	0.194***	0.197***	0.196***	0.196***
	(0.0717)	(0.0717)	(0.0717)	(0.0716)	(0.0718)	(0.0718)	(0.0717)	(0.0716)
L.NCD_CMBC	0.00319							
	(0.0130)							
L.NCD_CBB		0.00510						
		(0.0106)						
L.NCD_CZB			-0.0137					
			(0.0106)					
L.NCD_PAB				-0.0128				
				(0.0148)				
L.NCD_CGB					-0.00751			
					(0.0110)			
L.NCD_CEB						9.39e-05		
						(0.0112)		
L.NCD_HXB							-0.00282	
							(0.0109)	
L.NCD_EB								0.0160
								(0.0129)
Observations	748	748	748	748	748	748	748	748
R-squared	0.840	0.840	0.840	0.840	0.840	0.840	0.840	0.840

Appendix I – WMP Issuance in China

This appendix provides further background on the issuance of WMPs in China. As discussed in the main text, WMPs are the empirical counterpart to the shadow banking in our theoretical model. The model predicts that small banks (interbank price-takers) will be constrained by liquidity regulation before large banks (interbank price-setter) and engage more aggressively in shadow banking activities, both by offering higher returns (as measured by $\xi_j > \xi_k$) and by having a strict preference for booking the attracted funding off-balance-sheet (see Footnote 8 in the main text).

We use product-level WMP data from 2008 to 2014 from Wind. The benchmark sample includes all the WMPs issued by the Big Four and JSCBs. We first regress realized WMP returns on a dummy variable that equals one if the issuing bank is a member of the Big Four. We also include year and month dummies and control for WMP maturity. The results indicate that the realized returns on WMPs issued by the Big Four were on average 26 basis points lower than the realized returns on WMPs issued by the JSCBs. The difference is highly significant, with a standard error of 0.9 basis points. The gap increases to 37 basis points, with a standard error of 0.8, when the sample is extended to include WMPs issued by all small and medium-sized banks (SMBs). We then change the dependent variable to the realized returns relative to the expected floors advertised at issuance. The estimated coefficient on the Big Four dummy is 78 basis points, with a standard error of 2.3, suggesting that the Big Four were also more conservative than the JSCBs in the returns they advertised to investors. When we include all SMBs, the estimated coefficient on the Big Four dummy increases to 83 basis points, with a standard error of 1.7.

The data also corroborate the more aggressive issuance of off-balance-sheet WMPs by SMBs. Between 2008 and 2014, the JSCBs accounted for 73% of all new WMP batches and issued 57% of their batches without an explicit guarantee. The Big Four issued only 46% of their batches in this way. The gap in non-guaranteed intensity widens in the second half of the sample, with the JSCBs at 62% and the Big Four at 43%. These estimates are based on product counts since Wind does not yet have complete data on the total funds raised by each product. However, using data from CBRC and the annual reports of the Big Four, we estimate that SMBs (i.e., JSCBs and smaller) accounted for roughly 64% of non-guaranteed WMP balances outstanding at the end of 2013.[1] This conveys a consistent message with the batch statistics.

[1] The entire WMP balance in Bank of China's annual report is described in the notes as an unconsolidated balance yet the micro data in Wind includes several guaranteed batches for this bank that would not have matured by the end of 2013. We therefore remove Bank of China and rescale the other banks in the Big Four to back out our 64% estimate for small and medium-sized banks.

These differences between large and small banks arise because small banks are endogenously more constrained by liquidity regulation. Loan-to-deposit ratios based on *average balances during the year* in the early stages of CBRC's enforcement action provide a reasonable indicator of how constrained a bank would be by full implementation of the action. Average balance data is tabulated in the net interest analysis of bank annual reports, not the standard balance sheets that appear at the end of these reports.

Figure I.1 compares loan-to-deposit ratios based on average balance data (dashed lines) to those based on end-of-year data (solid lines). We plot ratios for the Big Four (blue) and the JSCBs (red) from 2005 to 2014. The shaded area is the interquartile range of the end-of-year ratios of the JSCBs. Data are from Bankscope and bank annual reports. Historical data for city and rural banks is spotty, especially when it comes to average balances, so these banks are excluded from the figure.[2]

It is clear from Figure I.1 that the JSCBs would have been more constrained than the Big Four as CBRC transitioned towards monitoring average balance ratios. First, there has never been a sizeable difference between the average balance and year-end loan-to-deposit ratios of the Big Four. In contrast, the JSCBs had consistently higher average balance ratios than year-end ratios prior to 2012, the first full year of average balance monitoring by CBRC. Second, the Big Four had both ratios comfortably below 75% before CBRC heralded the era of stricter and more frequent loan-to-deposit enforcement in 2008. This was not the case for the JSCBs who, as a group, were well above 75% based on average balance data and very close to 75% based on year-end data.[3]

For comparison, Figure I.2 plots the capital ratios of the Chinese banks from 2007 to 2014. China started to implement Basel III in 2013.[4] The required capital adequacy ratio (CAR) was increased from 8% in 2012 to 9% in 2014. The Tier-1 CAR was also increased from 4% to 7% over the same period. Neither the Big Four nor the JSCBs were constrained by capital requirements. Both CAR and Tier-1 CAR of the JSCBs exceeded the regulatory ratios by 2 or more percentage points every year from 2007 to 2014. The gap for the Big Four was even larger.

To provide formal evidence that off-balance-sheet issuance was driven by the bindingness

[2] One JSCB (Evergrowing Bank) is also excluded for similar reasons.

[3] We make two comments here. First, banks whose loan-to-deposit ratios are materially lower at the end of the year than on an average day during the year are window-dressing their year-end balance sheets. Hachem (2018) discusses the practices used in China before 2008 and why these practices could not be used to window-dress average balance ratios. Second, Figure I.1 shows that the loan-to-deposit ratio of the Big Four has increased towards 75% since the beginning of the enforcement. In Section 5.3, we demonstrate that much of this increase can be explained as a strategic response to increased competition from shadow banking.

[4] "Administrative Measures for the Capital of Commercial Banks," CBRC Document No. 1, 2012.

of liquidity regulation, we run panel regressions in Table I.1. We use bank-level data for each bank in the Big Four and the JSCBs. The dependent variable is the log of non-guaranteed WMP batches issued by bank i in year t scaled by the average balance of deposits at the bank in that year.[5] The main sample covers 2008 to 2010 inclusive. Recall that CBRC reached its final goal of average balance monitoring in mid-2011, making 2010 the last full year in which average balance ratios exhibit meaningful variation among constrained banks. In the first column of Table I.1, we regress the dependent variable on the loan-to-deposit ratio of bank i in year t, as measured using average balance data.[6] All columns include year fixed effects. We also control for the maturity of the non-guaranteed WMPs issued by bank i in year t. A bank that issues 3-month WMPs will issue twice as many WMP batches as a bank that issues 6-month WMPs to raise the same amount of funding over the course of a year. The bank with shorter-term WMPs will therefore have more batches, even if it is otherwise identical to the bank with longer-term WMPs. Including maturity as a regressor controls for this.

The results in the first column of Table I.1 confirm that banks with higher average balance ratios issued more non-guaranteed WMPs than banks with lower ratios. The second column shows that this finding is robust to controlling for the average return floor advertised by bank i when issuing non-guaranteed WMPs in year t. The third column shows that it is also robust to including bank fixed effects. In the fourth column, we extend the sample to 2014. The coefficient on the average balance ratio is still positive but its magnitude is roughly one-third of the magnitudes in the columns based on the main sample, and it is only statistically significant at the 10% level. The extended sample gives us more observations, and hence more degrees of freedom, to include the bank fixed effects. However, as noted earlier, the average balance ratio becomes a truncated indicator after 2010.

In the last two columns of Table I.1, we rerun the main sample regressions with the average balance ratio decomposed into two components: the regulated ratio of bank i in year t, as measured at the end of the year, and the degree of window-dressing by bank i in year t, as measured by the percent difference between the bank's average balance and regulated ratios.[7] The degree of window-dressing is the indication of constraint in this decomposition. The results in the last two columns corroborate what we found earlier:

[5] We use the indicator i here generically, i.e., not with reference to type.

[6] One may worry about reverse causality here. Specifically, the average balance ratio will decrease as the bank issues non-guaranteed WMPs to move some business off-balance-sheet. However, this implies a negative relationship between the dependent variable and the average balance ratio, which will bias the regression against us.

[7] We have to focus on the main sample in these columns as this is the sample where window-dressing is an observable (i.e., the average balance ratio becomes the regulated ratio once CBRC begins monitoring average balance ratios).

banks more constrained by CBRC's impending monitoring of average balance ratios issued more non-guaranteed WMPs than banks less constrained.

As a final exercise, we conduct Granger causality tests on total WMP issuance in Table I.2 and find that the WMPs issued by the Big Four were a response to the WMP activities of small and medium-sized banks (SMBs). We use differenced monthly data on WMP batches between January 2008 and September 2014 to run the tests. The Akaike Information Criterion (AIC) selects a VAR with 21 lags. As shown in Table I.2, the null hypothesis that WMP issuance by SMBs does not Granger-cause WMP issuance by the Big Four is rejected at 1% significance. The opposite hypothesis that WMP issuance by the Big Four does not Granger-cause WMP issuance by SMBs cannot be rejected at any reasonable level of significance. Using the Bayesian Information Criterion (BIC) to select the number of lags yields similar results, as do Granger causality tests based on other orders. The impetus for WMP activity in China is therefore coming from the SMBs, who are also the constrained banks and the banks more heavily involved in non-guaranteed issuance.

We conclude this appendix by discussing regional variation in financial sector growth after the tightening of liquidity regulation. The Wind database reports bank deposits, bank loans, and total assets of all financial institutions for each province in China. These aggregates sum over branch-level information that is not otherwise available. Provinces with higher initial loan-to-deposit ratios would be harder hit by the tightening of liquidity regulation, so the forces behind our credit boom result would predict an increase in shadow lending and an overall increase in credit in these provinces.

We first run a cross-provincial regression of the average annual growth rate of total financial institution assets from 2008 to 2014 on the loan-to-deposit ratio in 2007. The data are plotted in Panel A of Figure I.3. The slope of the fitted line is 0.12, with a standard error of 0.05. Next, we rerun the regression using the average annual growth rate of total financial institution assets less bank loans as the dependent variable. The data are plotted in Panel B of Figure I.3. The slope of the fitted line is 0.27, with a standard error of 0.07. In words, a 10 percentage point difference in loan-to-deposit ratios before CBRC's enforcement action is associated with a 2.7 percentage point difference in shadow loan growth and a 1.2 percentage point difference in total loan growth after the enforcement. Provincial statistics on the volume of shadow lending are not available, so we are using the difference between total assets of all financial institutions and bank loans as a proxy. Total lending, i.e., shadow lending plus traditional bank lending, is then proxied by the total assets metric. Our proxy for shadow lending is imperfect, so we do not want to overstate the provincial results. We only note that they are suggestive in the direction of our model. On this point, more WMP batches tend to be issued in provinces with higher loan-to-deposit ratios (Hachem (2018)), consistent with the result here that such provinces exhibit stronger shadow growth.

Table I.1

Non-Guaranteed WMP Issuance

	(1)	(2)	(3)	(4)	(5)	(6)
LDR	8.793***	9.764***	8.815**	2.720*		
	(1.719)	(2.623)	(3.283)	(1.381)		
Maturity	-0.145***	-0.138***	-0.181***	-0.045	-0.136***	-0.184***
	(0.016)	(0.018)	(0.042)	(0.045)	(0.022)	(0.052)
MinROR		-0.171	-0.135	-0.108	-0.171	-0.127
		(0.105)	(0.082)	(0.089)	(0.097)	(0.099)
WinDress					6.907*	6.179*
					(3.583)	(2.938)
RegRatio					10.676***	8.175
					(2.491)	(5.874)
Observations	41	31	31	79	31	31
Year Dummies	✓	✓	✓	✓	✓	✓
Bank Dummies	×	×	✓	✓	×	✓
R-squared	0.583	0.654	0.965	0.793	0.658	0.963

Notes: The dependent variable is the log of the total number of non-guaranteed WMPs issued by a bank in a year scaled by the average balance of deposits at the bank in that year. LDR is the loan-to-deposit ratio based on average balances of a bank in a year. Maturity and MinROR are respectively the average maturity and expected return floor on non-guaranteed WMPs issued by a bank in a year. WinDress is the percent difference between the average balance and year-end loan-to-deposit ratios of a bank in a year. RegRatio is the year-end ratio of a bank in a year. In all columns except (4), the sample period is 2008-2010. In column (4), the sample period is 2008-2014. Standard errors, clustered at the bank level, are in parentheses. ***p<0.01, **p<0.05, *p<0.1

Table I.2

Granger Causality Tests

H_0: SMB WMPs do not cause Big Four WMPs			
Criteria	Order	F-statistic	P-value
AIC	21	3.737	0.00
BIC	1	17.707	0.00
	3	7.095	0.00
	6	4.016	0.00
	9	2.295	0.02
H_0: Big Four WMPs do not cause SMB WMPs			
Criteria	Order	F-statistic	P-value
AIC	21	0.236	0.99
BIC	1	0.098	0.75
	3	0.966	0.41
	6	1.590	0.15
	9	0.492	0.88

Notes: We use monthly differenced data on WMP batches. AIC is the Akaike Information Criterion, which helps select the lag order of a VAR model for the Granger tests. BIC is the Bayesian Information Criterion. AIC usually over-estimates the order with positive probability, whereas BIC estimates the order consistently under fairly general conditions. Thus, BIC is typically used as the main selection criterion.

Figure I.1

Loan-to-Deposit Ratios

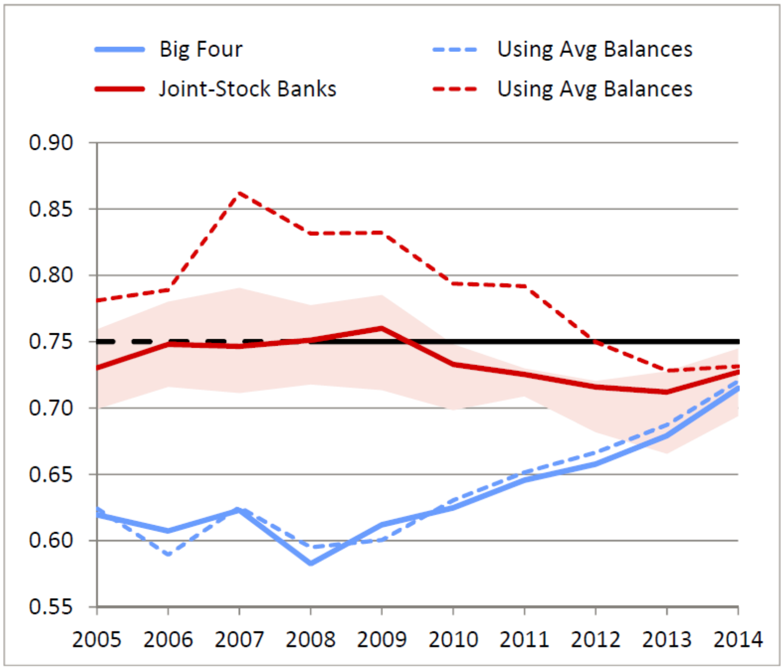


Figure I.2

Capital Adequacy Ratios

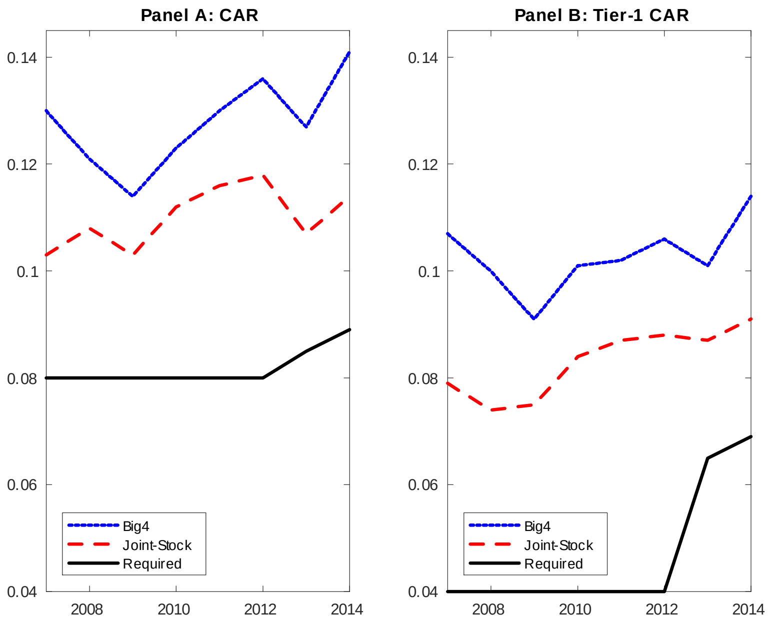
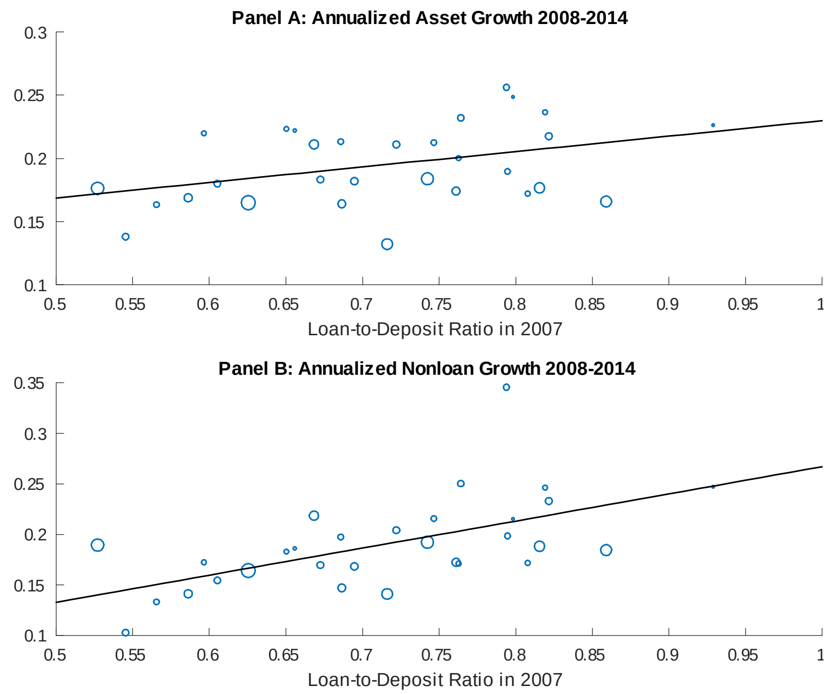


Figure I.3

Provincial Correlations



Appendix J – Sensitivity Analysis

This appendix conducts a sensitivity analysis of the credit boom in the calibrated model in response to tighter liquidity regulation. We change one parameter at a time, keeping all other parameters as in the baseline calibration of Section 5.2. To make the results comparable across experiments, we conduct local analysis. Specifically, we compute the change in the aggregate credit-to-savings ratio following an increase in the liquidity floor α from $\underline{\alpha}$ to $\underline{\alpha}+0.01$, where $\underline{\alpha}$ denotes the minimum $\alpha \geq 0.145$ at which $\xi_j > 0$. In the baseline calibration, $\underline{\alpha} = 0.145$ and a 1 percentage point increase in α from 0.145 to 0.155 increases the aggregate credit-to-savings ratio by 0.65 percentage points. In all the experiments we run, k remains unconstrained by the change in regulation, as in the baseline calibration.

Panel A of Figure J.1 plots the results for $\overline{r}_A$, the interest rate at which bank k lends on the interbank market. The baseline calibration normalizes $\overline{r}_A = 0$, which corresponds to an interbank rate of 2% in the data (see Section 5.2) and is thus depicted as $r_A = 0.02$ in Panel A. The sensitivity analysis on $\overline{r}_A$ demonstrates that our baseline credit boom is not driven by the normalization of $\overline{r}_A = 0$. For example, increasing $\overline{r}_A$ by 2 percentage points would still result in a nearly 0.5 percentage point increase in the aggregate credit-to-savings ratio following a 1 percentage point increase in α.[1]

As a separate but related sensitivity analysis on the modeling of $\overline{r}_A$, we can calibrate a version of the model with $\overline{r}_A$ endogenous. Recall that Appendix C microfounded $\overline{r}_A$ as the highest interbank rate that would leave the price-takers j with zero profit in state A. With shadow banking and operating costs, the expression becomes $\overline{r}_A\left(\lambda_j, \xi_j\right) = \frac{g(1-\lambda_j)+\lambda_j-1-\xi_j-\phi_j-\ell_j}{\theta_j^A-\lambda_j}$, where the parameter $\ell_j \geq 0$ is inversely related to the efficacy of bankruptcy courts; the higher is ℓ_j, the less profit can be seized by pushing borrowers to the brink of insolvency. Calibrating this extended model to start at $\overline{r}_A(\cdot) = 0$ when $\alpha = 0.145$ then increasing the liquidity floor to $\alpha = 0.25$ as in Section 5.3, we obtain a 5.2 percentage point increase in total credit. The same experiment with $\overline{r}_A = 0$ constant produced a 6.2 percentage point increase in total credit in Section 5.3. The credit boom is thus robust to endogenizing $\overline{r}_A$.

Returning to the local analysis of parameters in the baseline calibration, Panel B of Figure J.1 plots the results for z, which is the parameter that shifts the marginal return to investing in the long-term project, i.e., $g'(1-\lambda_i) = 1 + z - \gamma(1-\lambda_i)$. A higher value of z makes investing in the long-term project more attractive. The small, price-taking banks j then have even more incentive to increase ξ_j and shift funding into shadow banking, where liquidity regulation does not constrain how much they can invest. All else constant, this more

[1] Of course, there is a limit to how much we can increase $\overline{r}_A$ beyond what is shown in Panel A and still obtain an equilibrium with $r_B > 0$, which, as discussed in the main text, is the empirically relevant equilibrium (see Proposition 2 and the related discussion).

forcefully erodes the funding share x_k of the price-setter k and leads to a bigger credit boom. However, all else is not constant as k also wants to invest more in the long-term project. The price-setter thus chooses a higher ξ_k along with setting a higher average interbank rate $E(r)$ to temper ξ_j, both of which counteract the decrease in x_k and dampen the credit boom. The price-setter's response becomes stronger as z is increased further, leading to a hump-shaped effect of higher z on the size of the credit boom.

Panel C plots the results for δ_1, which governs the intensity of competition between j and k in Eq. (7). Higher δ_1 has two competing effects on the size of the credit boom. First, for a given $\xi_j > \xi_k$, higher δ_1 implies a bigger decline in k's funding share x_k and thus a bigger credit boom. Second, higher δ_1 compels k to choose a higher ξ_k, leading to a smaller credit boom. The first effect dominates at low values of δ_1 while the second dominates at high values. The result is a hump-shaped dependence of the size of the credit boom on δ_1.

In Panel D, we extend the model to allow for competition between the banks in type j. Specifically, we model the funding of any one bank j as

$$x_j = 1 - x_k^0 + \delta_1 \left(\overline{\xi_j} - \xi_k\right) + \delta_2 \left(\xi_j - \overline{\xi_j}\right)$$

where $\delta_2 > 0$. The baseline calibration corresponds to $\delta_2 = 0$. With $\delta_2 > 0$, an individual bank in type j can increase its funding x_j by increasing ξ_j relative to the average shadow banking action $\overline{\xi_j}$. In a symmetric equilibrium, $\xi_j = \overline{\xi_j}$ so x_k is still given by Eq. (7). Higher δ_2 implies more intense competition among small banks for funding. This bids up $\overline{\xi_j}$, triggering a competitive response from bank k to prevent further encroachment on its funding share x_k. In particular, k sets a higher $E(r)$ to stifle ξ_j and/or higher ξ_k, dampening the credit boom. Panel D illustrates the effect, but it is modest enough that a sizable credit boom remains.

Panel E plots the results for the external liquidity parameter L. With more external liquidity, bank k can lower r_B, the interest rate at which it borrows on the interbank market, without violating the aggregate feasibility condition in state B. This lowers $E(r)$, which facilitates higher ξ_j and leads to a bigger credit boom. Eventually though, i.e., for a large enough increase in ξ_j, it becomes profitable for k to recapture some of the lost funding x_k by also choosing higher ξ_k, dampening the credit boom. The first effect dominates at low values of L while the second effect dominates at high values. The result is a hump-shaped dependence of the size of the credit boom on L.

In Panel F, we extend the model to allow for interest-sensitive liquidity injections by the central bank. Specifically, the aggregate feasibility condition for each state $s \in \{A, B\}$ is now

$$\left(\lambda_j - \theta_j^s\right)(1 - x_k) + \left(\lambda_k - \theta_k^s\right) x_k + L + \psi r_s \geq 0$$

where $\psi > 0$.[2] The baseline calibration corresponds to $\psi = 0$. The effect of higher ψ on the size of the credit boom is qualitatively similar to the effect of higher L, i.e., hump-shaped, as both parameters increase liquidity on the interbank market for a given interest rate r_s.

We also conduct sensitivity analysis with respect to the price-setter's initial funding share. In the baseline calibration, we set $x_k^0 = 0.57$ so that the calibrated model delivers $x_k = 0.56$ at $\alpha = 0.145$, which was the deposit market share of the Big Four in 2007. Varying x_k^0 while keeping all other parameters as in the baseline calibration delivers $\xi_k > 0$ at $\underline{\alpha}$ if x_k^0 is sufficiently low, i.e., the price-setting bank will endogenously increase its initial funding share, undermining the experiment we want to conduct. A more appropriate exercise is therefore to recalibrate the model for different values of x_k^0 so that $\xi_k = 0$ at $\alpha = 0.145$, as in the benchmark model. We then report the change in the aggregate credit-to-savings ratio following a 1 percentage point increase in α.

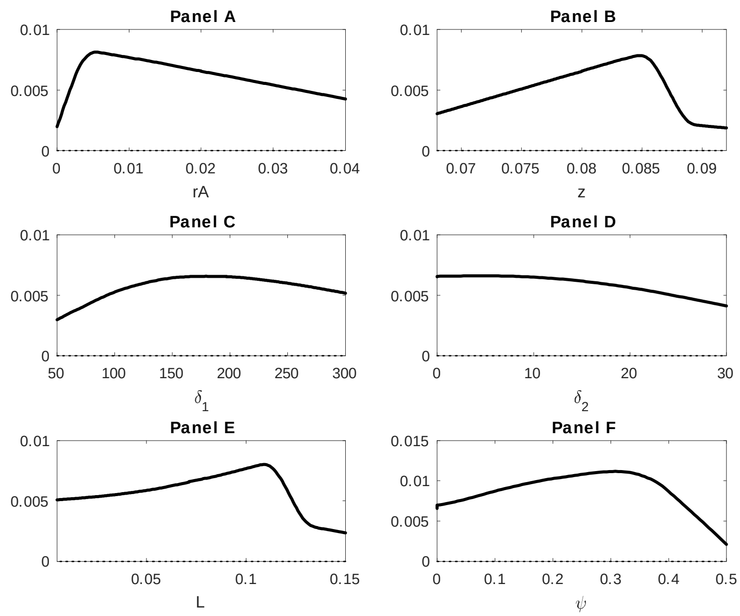
The results are presented in Table J.1. The first column corresponds to the baseline calibration. In the second column, we set $x_k^0 = 0.32$ and recalibrate the model to deliver $x_k = 0.31$ at $\alpha = 0.145$, which was the deposit market share of ICBC and CCB (the two biggest banks in the Big Four) in 2007. In the third column, we set $x_k^0 = 0.18$ and recalibrate the model to deliver $x_k = 0.17$ at $\alpha = 0.145$, which was the deposit market share of only ICBC (the biggest bank in the Big Four) in 2007. For both experiments, the other parameters are recalibrated to target the same empirical moments as in Section 5.2, with δ_1 recalibrated to target the same local percent change in x_k, i.e., from a 1 percentage point increase in α, rather than the level in 2014. This recalibration recovers lower values of δ_1 as x_k^0 is decreased, i.e., a smaller bank k experiences less deposit outflow to its competitors j for the same spread $\xi_j - \xi_k$.

We can see from Table J.1 that calibrating the model to a smaller price-setting bank dampens the size of the credit boom. However, the effect remains quantitatively important, decreasing less than proportionally with the initial size of the price-setter. To this point, Table J.1 shows that the average interbank rate increases more aggressively with liquidity regulation when the price-setter is smaller. This permits a larger increase in the price-setter's loan-to-deposit ratio (see Section 3.5), which helps sustain the credit boom.

[2] Since we deduct 2% from all interest rates in the calibration, this equation when compared to Eq. (13) implicitly has an interest target rate of 2%.

Figure J.1: Effect of $\Delta\alpha = 0.01$

Δ Credit-to-Savings Ratio



Notes: The experiments in this figure vary one parameter at a time, keeping all other parameters as in the baseline calibration.

Table J.1: Effect of $\Delta\alpha = 0.01$

	$x_k^0 = 0.57$	$x_k^0 = 0.32$	$x_k^0 = 0.18$
Δ Credit-to-Savings Ratio	0.65 pp	0.46 pp	0.29 pp
Δ Avg. Interbank Rate	9 bps	14 bps	16 bps
Δ Loan-to-Deposit Ratio of k	1 pp	2.4 pp	4.9 pp

Notes: The experiments in this table recalibrate the model to deliver different initial funding shares. The first column corresponds to the baseline calibration. In each column, the policy experiment is a 1 pp increase in α.

Appendix K – Estimation Procedure

Let $m = 1, ..., 4$ index the empirical moments to be matched. The four moments are the four correlations in Table 3.

1. Bootstrap: Let N denote the total number of random samples generated by bootstrap. We set $N = 500$. Denote by $y_{m,n}$ the m^{th} moment in the n^{th} sample. We will target $\frac{1}{N}\sum_{n}^{N} y_{m,n}$, the m^{th} moment averaged across N samples.

2. Denote by Ω the vector of parameters to be estimated. Given Ω, we can simulate the model to generate the moments $y_m(\Omega)$. Denote by $\varepsilon_{m,n} = y_m(\Omega) - y_{m,n}$ the residual for moment m in sample n. Define the weighting matrix $(M \times M)$ as:

$$W = \frac{1}{N}\sum_{n}^{N} \varepsilon_{m,n}\varepsilon_{m,n}^{T}$$

3. Minimizing the weighted sum of the distance between the empirical and simulated moments:

$$\widehat{\Omega} = \arg\min_{\Omega} \ell(\Omega)' W^{-1} \ell(\Omega)$$

   where $\ell(\Omega)$ is a vector with M elements and $\ell_m(\Omega) = y_m(\Omega) - \frac{1}{N}\sum_{n}^{N} y_{m,n}$.

4. We use two-step Simulated Method of Moments. We set W to the identity matrix in the first step and use the variance-covariance matrix of the residuals from the first-step as the weighting matrix for the second-step estimation.

5. Repeat the above exercise 100 times to calculate the standard errors of the estimated parameters.